

CHAPTER 7

Properties of shapes

In this chapter you will:

- Calculate the area and perimeter of 2 dimensional shapes
- Calculate the volume and surface area of 3 dimensional shapes
- Calculate the sizes of angles
- Calculate the sizes of angles using the straight line relationships
- Name an angle according to its position
- Use parallel lines to calculate the sizes of angles
- Prove lines parallel
- Calculate the sum and the size of the interior angles of polygons
- Distinguish between congruent and similar polygons
- Name triangles according to the length of their sides
- Calculate the sizes of angles in isosceles triangles
- Name congruent triangles
- Prove triangles congruent
- Prove triangles similar
- Use the definitions of quadrilaterals to look at the relationship between them

This chapter covers material from Topic 3: Space, Shape and Orientation

SUBJECT OUTCOME 3.1:

Describe, represent, analyse and explain properties of shape in two-and three-dimensional space with justification

Learning Outcome 2: Investigate and generalise formula for calculating the volume and surface area of right prisms and cylinders.

SUBJECT OUTCOME 3.2:

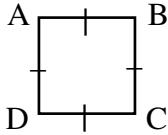
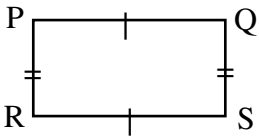
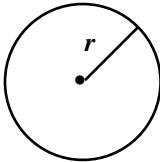
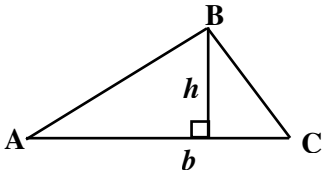
Describe and represent the properties of geometric shapes

Learning Outcome 1: Investigate polygons through Euclidian, co-ordinate and/or transformation geometry

Learning Outcome 2: Generalise the properties of different polygons

Learning Outcome 3: Expose alternate definitions of polygons

§ 7.1 AREA AND PERIMETERS OF 2 DIMENSIONAL FIGURES

NAME	SHAPE	FORMULA FOR AREA	FORMULA FOR PERIMETER
square		Area ABCD = side ² = AB ²	Perimeter = 4 × side = 4AB
rectangle		Area PQRS = length × breadth = PQ × QS	Perimeter = 2 length + 2 breadth = 2PQ + 2QS
circle		Area = πr^2 • r is the radius • d (diameter) = $2r$	Circumference = $2\pi r$
triangle		Area Δ ABC = $\frac{1}{2} \times b \times h$ • b is the length of the base • h is the length of the perpendicular height	Perimeter = AB + BC + AC

EXAMPLE	SOLUTION	
Find the area (A) and perimeter (P) of:		
1) a square: side = 4 cm	$A = 4^2 = 16 \text{ cm}^2$	$P = 4 \times 4 = 16 \text{ cm}$
2) a rectangle: length = 3 cm and breadth = 6 cm	$A = 3 \times 6 = 18 \text{ cm}^2$	$P = 2(3) + 2(6)$ $= 6 + 12 = 18 \text{ cm}$
3) a circle: diameter = 6 cm	$A = \pi \times 3^2 = 28,3 \text{ cm}^2$	$C = 2 \times \pi \times 3 = 18,8 \text{ cm}$
4) a triangle: base = 8 cm, height = 3 cm, sides = 5 cm	$A = \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2$	$P = 8 + 5 + 5 = 18 \text{ cm}$

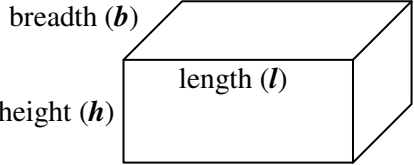
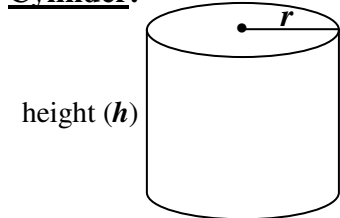
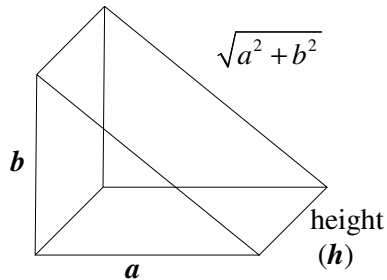
Exercise 7.1

Find a) the area (A) and b) perimeter (P), correct to 1 decimal place, of:

	(a) Area	(b) Perimeter
1) a square: side = 8 cm		
2) a rectangle: length = 7 cm and width = 5 cm		
3) a circle: diameter = 8 cm		
4) a triangle: base = 24 cm, height = 5 cm, other two sides = 13 cm		

§ 7.2 VOLUME AND SURFACE AREA OF 3 DIMENSIONAL SHAPES

✦ To find the volume of a prism, multiply the area of the base by the height.

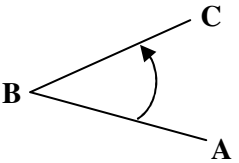
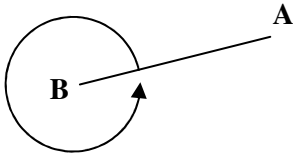
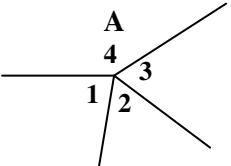
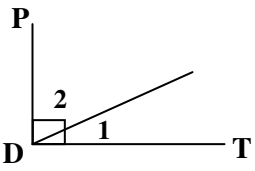
3-dimensional figure	Volume (V)	Surface area (SA)
<u>Rectangular prism:</u> 	The base is a rectangle. Area of base = $l \times b$ $V = l \times b \times h$	6 rectangular faces $SA = 2lb + 2lh + 2bh$
<u>Cylinder:</u> 	The base is a circle. Area of base = πr^2 $V = \pi r^2 h$	2 circular faces and a curved surface. $SA = 2\pi r^2 + 2\pi rh$
<u>Triangular prism:</u> 	The base is a right-angled triangle. Area of base $= \frac{1}{2} \times a \times b$ $V = \frac{1}{2} \times a \times b \times h$	2 triangular faces and 3 rectangular faces. Area of 2 triangular faces $= 2\left(\frac{1}{2}ab\right) = ab$ Surface Area $= ab + bh + ah + \sqrt{a^2 + b^2} \cdot h$

Exercise 7.2

Find (a) the volume (V) and (b) surface area (SA), correct to 1 decimal place, of a:

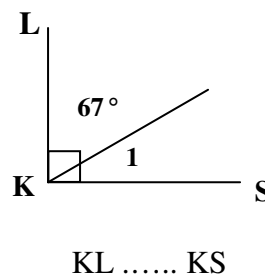
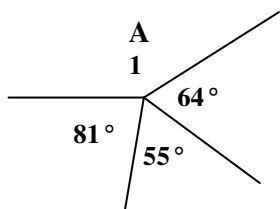
	(a) Volume	(b) Surface Area
1) A rectangular prism having length = 14 cm, breadth = 5 cm and height = 4 cm		
2) A closed cylinder of diameter = 12 cm and height = 20 cm		
3) A right-angled triangular prism with side = 5cm, side = 12 cm (not the hypotenuse) and height = 10 cm		

§ 7.3 ANGLES

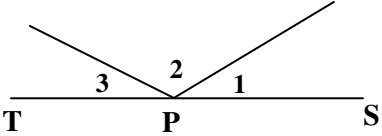
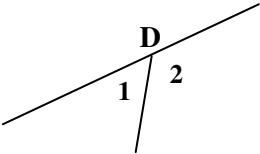
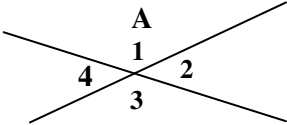
	- Turning BA about B to position BC forms an angle. We name the angle $\hat{A}BC$ or $\hat{B}$. It is measured in degrees. - B is the vertex of the angle - BC and BA are arms of the angle
	- Arm BA is rotated about B through a full circle, back to its original position - the size of the angle formed is 1 revolution = 360°
	Angles about point A add to 360° $\hat{A}_1 + \hat{A}_2 + \hat{A}_3 + \hat{A}_4 = 360^\circ$
	- Angles adding to 90° are called complementary angles $\hat{D}_1$ and $\hat{D}_2$ are complementary since $\hat{D}_1 + \hat{D}_2 = 90^\circ$ $\hat{D}_1$ is the complement of $\hat{D}_2$ - DP and DT are perpendicular lines, written $DP \perp DT$

Exercise 7.3

- 1) What is the complement of 54° ?
- 2) What is the size of 1 revolution?
- 3) What do the angles about a point add up to?
- 4)
 - a) Calculate the size of $\hat{A}_1$.
 - b) Calculate the size of $\hat{K}_1$.



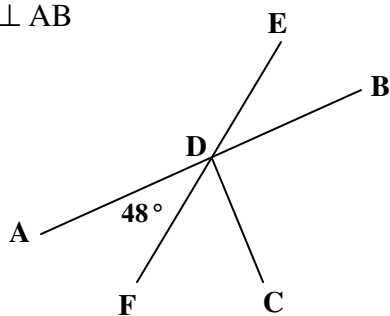
§ 7.4 STRAIGHT LINE RELATIONSHIPS

	TPS is a straight line. • Adjacent angles on a line add to 180° • $\hat{P}_1 + \hat{P}_2 + \hat{P}_3 = 180^\circ$
	• Angles adding to 180° are supplementary angles • $\hat{D}_1$ and $\hat{D}_2$ are supplementary since $\hat{D}_1 + \hat{D}_2 = 180^\circ$ • $\hat{D}_1$ is the supplement of $\hat{D}_2$
	• Vertically opposite angles are equal in size • $\hat{A}_1 = \hat{A}_3$ and $\hat{A}_2 = \hat{A}_4$

Exercise 7.4

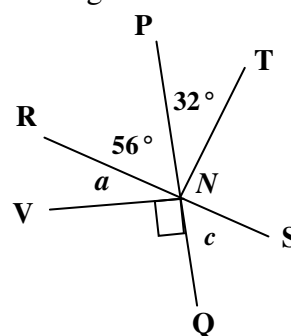
- 1) What is the supplement of 98° ?
- 2) What is the supplement of 129° ?
- 3) What is the sum of adjacent angles on a line?
- 4) What is the size of an angle vertically opposite to an angle of 136° ?
- 5) Calculate:

- a) FE and AB are straight lines.
 $CD \perp AB$



$\hat{CDB} = \dots\dots\dots$
 $\hat{EDB} = \dots\dots\dots$
 $\hat{CDE} = \dots\dots\dots$
 $\hat{CDF} = \dots\dots\dots$
 $\hat{FDB} = \dots\dots\dots$
 $\hat{ADE} = \dots\dots\dots$

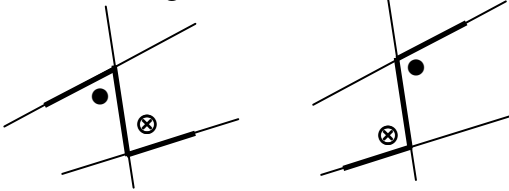
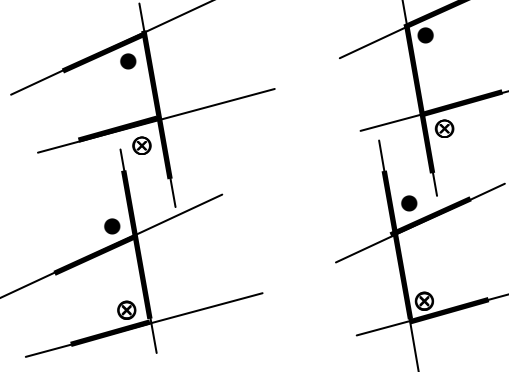
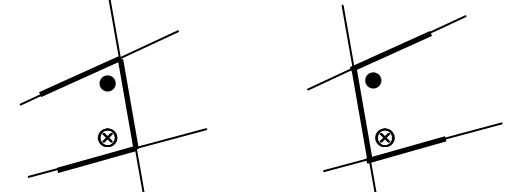
- b) PQ and RS are straight lines.



$c = \dots\dots\dots$
 $a = \dots\dots\dots$
 $\hat{TNS} = \dots\dots\dots$
 $\hat{RNT} = \dots\dots\dots$
 $\hat{VNT} = \dots\dots\dots$
 $\text{Reflex } \hat{RNQ} = \dots\dots\dots$
 $\hat{SNV} = \dots\dots\dots$

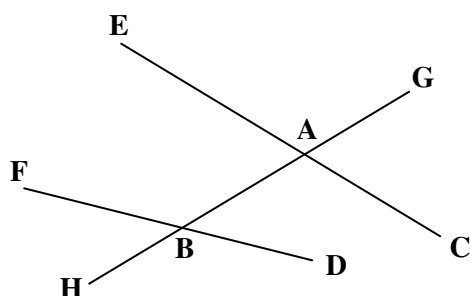
§ 7.5 ANGLE POSITION

✦ In each sketch below, two lines are cut by a transversal. The marked pairs of angles have a name according to their position. The angle pairs are alternate or corresponding or co-interior.

1) Alternate angles: 	• Alternate angles are recognised by their position in a "Z" shape • The "Z" shape may be back to front • There are 2 pairs of alternate angles
2) Corresponding angles: 	• Corresponding angles lie in an "F" shape • The "F" shape may be back to front or upside down • There are 4 pairs of corresponding angles
3) Co-interior angles: 	• Co-interior angles lie within a "C" shape • The "C" shape may be back to front • There are two pairs of co-interior angles

Exercise 7.5

1) Use the sketch to write down the names of:



a) 4 pairs of corresponding angles:

b) 2 pairs of co-interior angles:

c) 2 pairs of alternate angles:

2) a) The shape associated with corresponding angles is

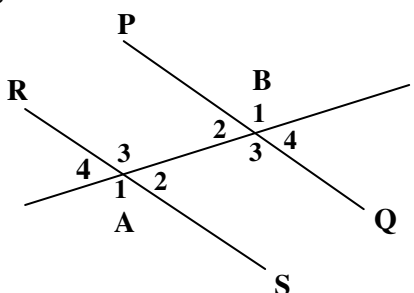
b) The shape associated with alternate angles is

c) The shape associated with co-interior angles is

§ 7.6 PARALLEL LINES

When two lines are parallel and are cut by a transversal, then:

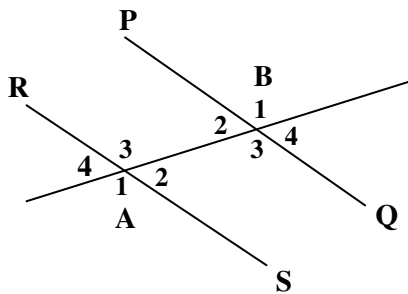
- ✦ The alternate angles are equal
- ✦ The corresponding angles are equal
- ✦ The co-interior angles add to 180° (they are supplementary)

EXAMPLE	SOLUTION
<u>Given:</u> $PQ \parallel RS$ and $\hat{B}_4 = 72^\circ$ Calculate the marked angles. Give reasons for your answers.  <u>Note:</u> There are a number of different ways to find the size of an angle.	$\hat{B}_4 = \hat{B}_2 = 72^\circ$ vertically opposite angles $\hat{B}_2 + \hat{A}_3 = 180^\circ$ co-interior angles $\hat{A}_3 = 108^\circ$ $\hat{B}_1 = 180^\circ - 72^\circ$ adjacent angles on a line $= 108^\circ$ $\hat{B}_1 = \hat{B}_3 = 108^\circ$ vertically opposite angles $\hat{B}_2 = \hat{A}_2 = 72^\circ$ alternate angles $\hat{B}_3 = \hat{A}_1 = 108^\circ$ corresponding angles $\hat{A}_2 = \hat{A}_4$ vertically opposite angles

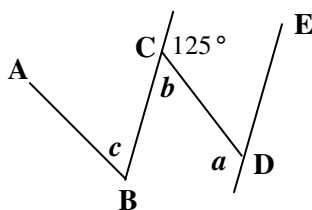
Exercise 7.6

Calculate the marked angles. Give reasons for your answers.

- 1) Given: $PQ \parallel RS$ and $\hat{A}_1 = 112^\circ$



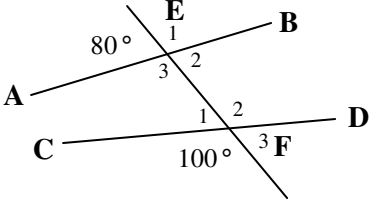
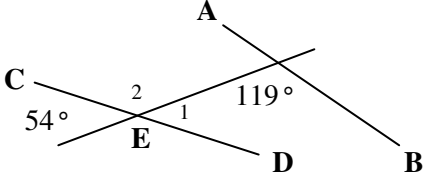
- 2) Given: $AB \parallel CD$ and $CB \parallel ED$



§ 7.7 PROVING LINES PARALLEL

If two lines are cut by a transversal, and

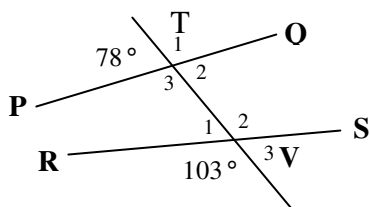
- ✦ one pair of alternate angles is equal **OR**
 - ✦ one pair of corresponding angles is equal **OR**
 - ✦ one pair of co-interior angles adds to 180° (they are supplementary)
- then the lines are parallel.

EXAMPLE	SOLUTION
Is $AB \parallel CD$? Give a reason for your answer. 1) 	Yes. $\hat{F}_1 = 80^\circ$ angles on a line $AB \parallel CD$ corresponding angles equal
2) 	No. $\hat{E}_1 = 54^\circ$ vertically opposite angles $54^\circ + 119^\circ = 173^\circ \neq 180^\circ$ co-interior angles are not supplementary.

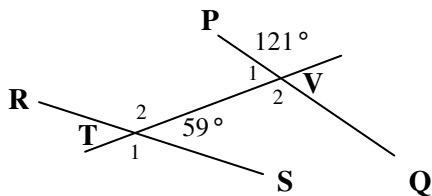
Exercise 7.7

Is $PQ \parallel RS$? Give a reason for your answer.

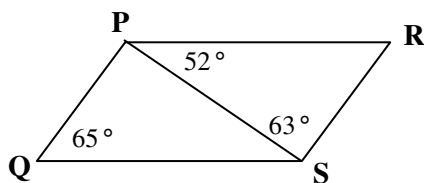
1)



2)

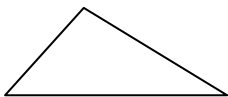
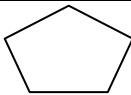
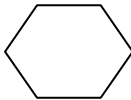


3) Given: $PR \parallel QS$



§ 7.8 POLYGONS

- ✦ A **polygon** is a closed figure having three or more sides which are straight lines.
- ✦ A **regular polygon** has sides equal in length and angles equal in size.

Name	Number of sides	Polygon
triangle	3	
quadrilateral	4	
pentagon	5	
hexagon	6	

Note:

- The **sum of the interior angles** of a polygon of n sides is $180^\circ \times (n - 2)$
- The **size of an interior angle** of a regular polygon of n sides is $\frac{180^\circ(n - 2)}{n}$
- Other polygons are: heptagon (7 sides), octagon (8 sides), nonagon (9 sides), decagon (10 sides)

EXAMPLE	SOLUTION
1) Calculate the sum of the interior angles of a regular pentagon	A pentagon has 5 sides, so $n = 5$. Sum of interior angles = $180^\circ (5 - 2) = 180^\circ (3) = 540^\circ$
2) Calculate the size of each interior angle.	The size of each interior angle = $540^\circ \div 5 = 108^\circ$.

Exercise 7.8

Calculate:

- a) the sum of the interior angles of the regular polygon
 - b) the size of each interior angle.
- 1) Hexagon
 - 2) Octagon
 - 3) Triangle
 - 4) Quadrilateral
 - 5) Decagon

§ 7.9 SIMILAR AND CONGRUENT POLYGONS

Congruent polygons:

- ✦ Congruent polygons are **identical** to each other. Their:
 - corresponding sides are equal in length and
 - corresponding angles are equal in size
- ✦ A congruent polygon is obtained when the given polygon is translated, reflected or rotated.

EXAMPLE	SOLUTION
	$\triangle DEF$ has been reflected to form $\triangle BAC$, but its sides and angles are unchanged. $\triangle DEF \equiv \triangle BAC$

Similar polygons:

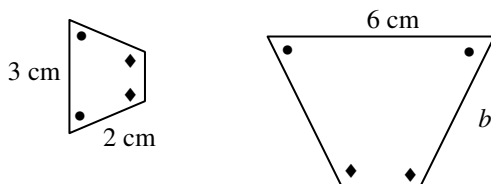
- ✦ Polygons are similar if:
 - they are equiangular (angles equal in size) **and**
 - their corresponding sides are in proportion.
- ✦ Similar polygons are obtained when a given polygon is enlarged or reduced.

EXAMPLE	SOLUTION
	ABCD and EFGH are equiangular The sides of ABCD and EFGH are in the proportion 2 : 1 So $ABCD \sim EFGH$

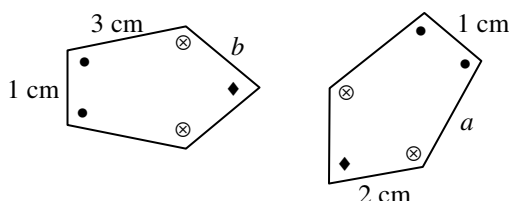
Exercise 7.9

- 1) Are the following figures congruent or only similar?
- 2) Calculate the lengths of the marked sides.

a)

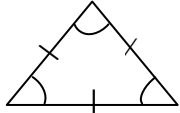
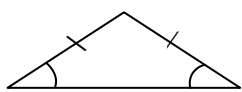


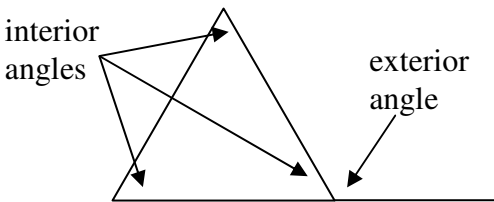
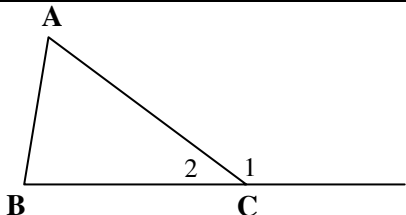
b)



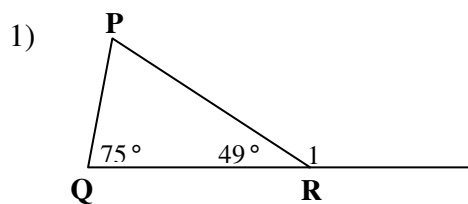
§ 7.10 TRIANGLES

✦ A triangle is a three sided polygon.

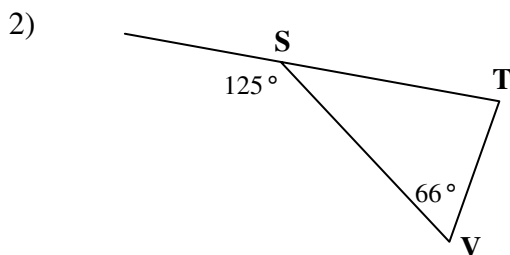
Type	Figure	Properties
Scalene triangle		- No equal sides - No equal angles
Equilateral triangle		- Three equal sides - Three equal angles - All angles 60°
Isosceles triangle		- Two equal sides - Two equal angles

 interior angles exterior angle	✦ Every triangle has three interior angles. 'interior' means 'inside'. ✦ The exterior angle of a triangle lies between one side and an extended side of the triangle.
 A B C 2 1	✦ The exterior angle of a triangle is equal to the sum of the two interior opposite angles. $\hat{C}_1 = \hat{A} + \hat{B}$ ✦ The 3 interior angles of a triangle add to 180°. $\hat{A} + \hat{B} + \hat{C}_2 = 180^\circ$

Exercise 7.10



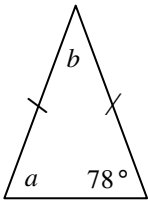
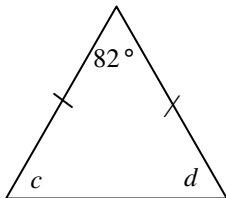
Calculate $\hat{P}$ and $\hat{R}_1$



Calculate $\hat{T}$ and $\hat{TSV}$.

§ 7.11 ISOSCELES TRIANGLES

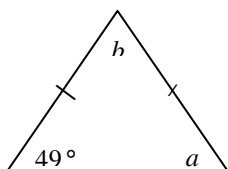
- ✦ If **two sides of a triangle are equal in length**, the triangle is isosceles, and the angles opposite those sides are equal in size
- ✦ If **two angles of a triangle are equal in size**, the triangle is isosceles, and the sides opposite those angles are equal in length

EXAMPLE	SOLUTION
Calculate the size of the marked angles: 1) 	$a = 78^\circ$ angles opposite equal sides $b + 78^\circ + 78^\circ = 180^\circ$ angle sum of triangle $b + 156^\circ = 180^\circ$ $b + 156^\circ - 156^\circ = 180^\circ - 156^\circ$ $b = 24^\circ$
2) 	$82^\circ + c + d = 180^\circ$ angle sum of triangle $82^\circ + c + d - 82^\circ = 180^\circ - 82^\circ$ $c + d = 98^\circ$ $c = d$ angles opposite equal sides $c = d = 49^\circ$

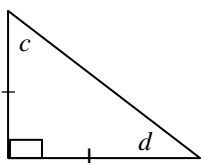
Exercise 7.11

Calculate the size of the angles marked in the figure. Give reasons for your answer.

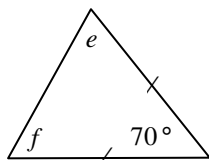
1)



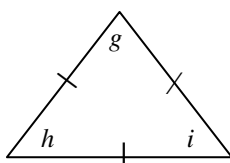
2)



3)

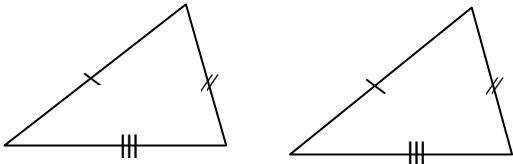
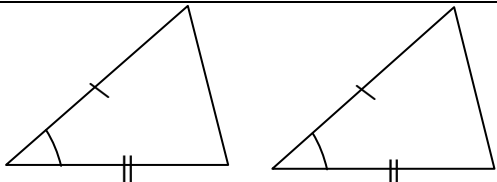
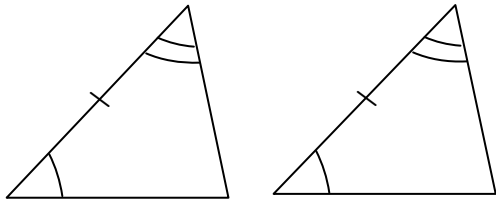
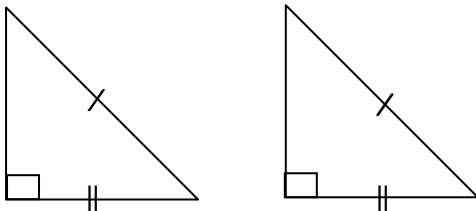


4)



§ 7.12 CONGRUENT TRIANGLES

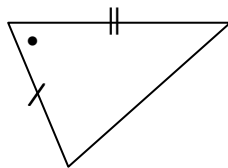
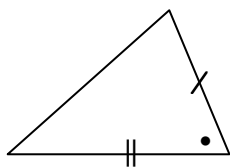
✦ Two triangles are congruent if **one** of these four conditions is true:

1)		- Three sides of one triangle equal in length to three sides of the other triangle - Notation: SSS
2)		- Two sides of one triangle equal to two sides of the other AND - The included angles equal in size - Notation: SAS
3)		- Two angles of one triangle equal in size to two angles of the other triangle AND - One side of one triangle equal in length to the corresponding side of the other - Notation: SAA
4)		- Both triangles are right-angled AND - Hypotenuses are equal AND - One pair of corresponding sides is equal - Notation: RHS

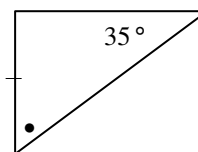
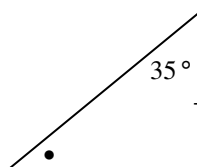
Exercise 7.12

1) Are the following pairs of triangles congruent? If yes, give a reason for your answer.

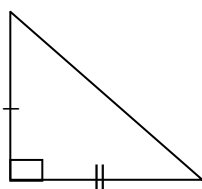
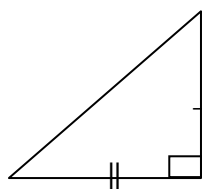
a)



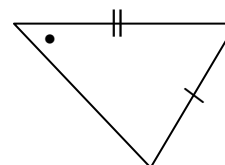
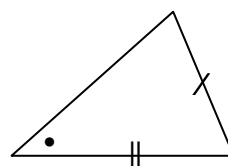
b)



c)

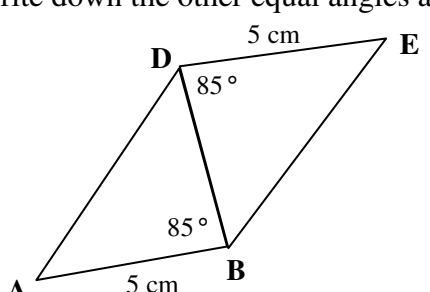


d)



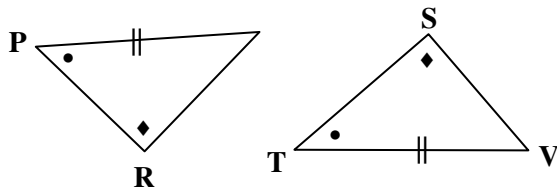
2) Is AAA a condition for congruency?

§ 7.13 PROVING TRIANGLES CONGRUENT

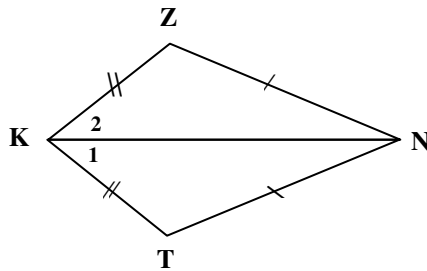
EXAMPLE	SOLUTION
a) Prove: $\triangle DEB \equiv \triangle BAD$ b) Write down the other equal angles and sides 	a) $\hat{E}DB = \hat{D}BA = 85^\circ$ (given) $DE = AB = 5 \text{ cm}$ (given) $DB = DB$ (common side) $\triangle DEB \equiv \triangle BAD$ (SAS) b) $BE = AD$ (congruency) $\hat{D}BE = \hat{B}DA$ (congruency) $\hat{E} = \hat{A}$ (congruency)

Exercise 7.13

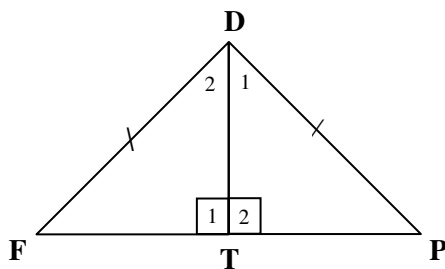
- 1) a) Prove: $\triangle PQR \equiv \triangle STV$
 b) Write down other equal angles and sides



- 2) a) Prove $\triangle KZN \equiv \triangle KTN$ b) $\hat{K}_1 = \hat{K}_2$

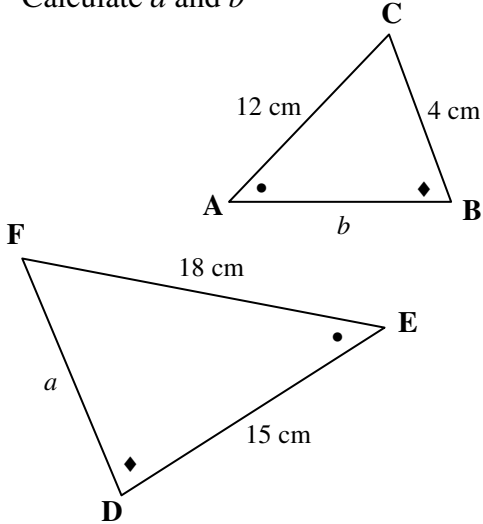


- 3) a) Prove $\triangle DTF \equiv \triangle DTP$
 b) $\hat{D}_2 = \hat{D}_1$ and $\hat{F} = \hat{P}$



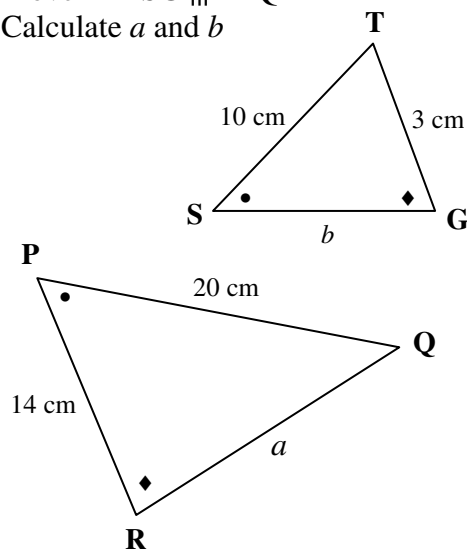
§ 7.14 SIMILAR TRIANGLES

- ✦ If two triangles are **equiangular**, their corresponding sides are in proportion.
- ✦ If two triangles have their **corresponding sides in proportion**, they are equiangular.
- ✦ 'Equiangular' means 'equal angles' which means that the three angles of the one triangle are equal in size to the three angles of the other triangle
- ✦ Triangles are **similar** if:
 - they are equiangular **OR**
 - the lengths of their corresponding sides are in proportion

EXAMPLE	SOLUTION
1) Prove $\triangle ABC \sim \triangle DEF$ 2) Calculate a and b 	$\hat{A} = \hat{E}$ $\hat{B} = \hat{D}$ $\hat{C} = \hat{F}$ (angle sum of triangle) $\triangle ABC \sim \triangle EDF$ (AAA) $\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$ $\frac{b}{15} = \frac{12}{18} = \frac{4}{a}$ $\frac{b}{15} \times 15 = \frac{12}{18} \times 15$ $b = 10 \text{ cm}$ $\frac{12}{18} \times a \times \frac{18}{12} = \frac{4}{a} \times a \times \frac{18}{12}$ $a = 6 \text{ cm}$

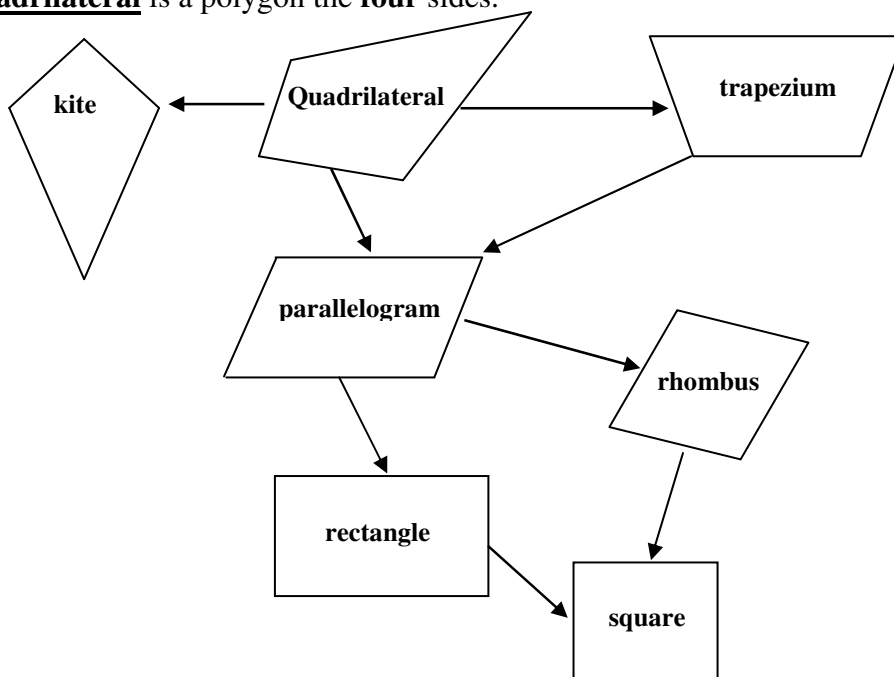
Exercise 7.14

- 1) Prove $\triangle TSG \sim \triangle QPR$
 2) Calculate a and b



§ 7.15 TYPES OF QUADRILATERALS

A **quadrilateral** is a polygon the **four** sides.



The chart above helps to understand the definitions of the different quadrilaterals.

- ✦ A **kite** is a quadrilateral with two pairs of adjacent sides equal.
- ✦ A **trapezium** is a quadrilateral with one pair of parallel sides.
- ✦ A **parallelogram** is a quadrilateral with two pairs of parallel sides.
- ✦ A **rectangle** is a parallelogram with right angles.
- ✦ A **square** is a rectangle with four equal sides.
- ✦ A **rhombus** is a parallelogram with four equal sides.

Exercise 7.15

- 1) Define a square in terms of a rhombus.
.....
- 2) Define a parallelogram in terms of a trapezium.
.....
- 3) Answer Sometimes (S) or Always (A):

a) A rhombus is a square	b) A square is a rectangle
c) A trapezium is a parallelogram	d) A quadrilateral is a trapezium
e) A kite is a quadrilateral	f) All parallelograms are quadrilaterals
g) A rhombus is a rectangle	h) A kite is a parallelogram
i) A square is a rhombus	j) All squares are parallelograms

CHAPTER 8

Analytical and transformation geometry

In this chapter you will:

- Use the formula for the gradient of a line and the distance formula
- Use the formula for the midpoint of a line segment
- Find the equation of a line given the co-ordinates of two points on the line
- Change the form of the equation of a straight line
- Translate polygons
- Reflect a polygon about the x -axis and about the y -axis.
- Reflect a polygon about the line $y = x$
- Reflect a polygon about the line $y = -x$

This chapter covers material from Topic 3: Space, Shape and Orientation

SUBJECT OUTCOME 3.2:

Describe and represent the properties of geometric shapes

Learning Outcome 1: Investigate polygons through Euclidian, co-ordinate and/or transformation geometry

SUBJECT OUTCOME 3.3

Represent geometric figures on a Cartesian co-ordinate system

Learning Outcome 1: Plot points, lines and polygons on a Cartesian plane

Learning Outcome 2: Calculate the distance between two points

Learning Outcome 3: Calculate the gradient of a line segment joining two points

Learning Outcome 4: Calculate the midpoint of a line segment joining two points on a graph

Learning Outcome 5: Estimate distance, slope and midpoint from a graphical representation of a straight line

SUBJECT OUTCOME 3.4

Solve problems by constructing and interpreting geometrical and trigonometric models

Learning Outcome 1: Use transformation geometry to translate p units horizontally and q units vertically

Learning Outcome 2: Use transformation geometry to reflect graphs about the x -axis, the y -axis and the line $y = x$

§ 8.1 GRADIENT AND DISTANCE FORMULAE

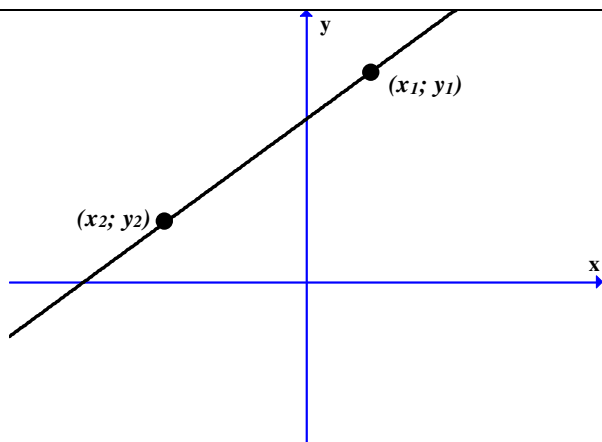
- ✧ The **gradient** of a straight line passing through the points $(x_1; y_1)$ and $(x_2; y_2)$ is

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

- ✧ The **distance** between the points $(x_1; y_1)$ and $(x_2; y_2)$ is $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$.

Note:

- Points that lie on the same straight line are called **collinear** points
- The gradient is the same between any two points on a straight line



EXAMPLE	SOLUTION
1) Determine whether the points A(1; -1), B(-1; -5) and C(2; 1) are collinear.	$\text{Gradient}_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-1)}{-1 - 1} = \frac{-4}{-2} = 2$ $\text{Gradient}_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-5)}{2 - (-1)} = \frac{1 + 5}{2 + 1} = \frac{6}{3} = 2$ $\text{Gradient}_{AB} = \text{Gradient}_{BC}$ $\therefore$ A, B and C are collinear (gradients are the same)
2) Prove that $\triangle PQR$ with vertices P(1; 1), Q(1; 4) and R(5; 1) is right-angled. Where is the right-angle in the triangle?	$\text{Distance}_{PQ} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $= \sqrt{(4 - 1)^2 + (1 - 1)^2} = \sqrt{3^2} = 3$ $\text{Distance}_{QR} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $= \sqrt{(1 - 4)^2 + (5 - 1)^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ $\text{Distance}_{PR} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $= \sqrt{(1 - 1)^2 + (5 - 1)^2} = \sqrt{4^2} = 4$ • $PQ : PR : QR = 3 : 4 : 5$, so the triangle is right-angled. • QR is the hypotenuse, so the right angle is at P.

Exercise 8.1

- 1) Prove that $\triangle ABC$ having vertices A(2; 4), B(0; 3) and C(6; -4) is right-angled. Where is the right-angle in the triangle?
- 2) Determine whether the points P(2; -4), R(-2; -8) and S(0; -2) are collinear.

§ 8.2 FORMULA FOR THE MIDPOINT OF A LINE SEGMENT

✧ The formula for finding the co-ordinates of the midpoint of a line segment is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \text{ where } (x_1; y_1) \text{ and } (x_2; y_2) \text{ are points on the graph}$$

EXAMPLE	SOLUTION
1) Calculate the mid-point C of the line joining A(-2; -3) to B(4; -9).	$C\left(\frac{-2+4}{2}; \frac{-3+(-9)}{2}\right) = \left(\frac{2}{2}; \frac{-12}{2}\right) = (1; -6)$
2) Calculate a and b if R(4; -3) is the mid-point of the line joining P(a ; -8) to Q(10; b).	$4 = \frac{a+10}{2} \qquad -3 = \frac{-8+b}{2}$ $4 \times 2 = \frac{a+10}{2} \times 2 \qquad -3 \times 2 = \frac{-8+b}{2} \times 2$ $8 = a + 10 \qquad -6 = -8 + b$ $8 - 10 = a + 10 - 10 \qquad -6 + 8 = -8 + b + 8$ $-2 = a \qquad 2 = b$

Exercise 8.2

Calculate x and y :

1) if P(x ; y) is the mid-point of the line joining Q(-1; 4) to R(5; -6)

2) if N(0; 2) is the mid-point of the line joining C(x ; y) to D(6; 0)

3) if R(6; -2) is the mid-point of the line joining P(-3; y) to Q(x ; -8)

§ 8.3 FINDING THE EQUATION OF A STRAIGHT LINE

✧ Suppose a straight line passes through the points $(x; y)$ and $(x_1; y_1)$

The gradient of the line is $a = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\text{Then } a(x - x_1) = \frac{y - y_1}{(x - x_1)} \times (x - x_1)$$

So $a(x - x_1) = y - y_1$ is the equation of a straight line graph.

EXAMPLE	SOLUTION
1) Find the equation of the line passing through the points D(-3; -7) and F(-5; 3). Note: It does not matter which point is used to substitute into the equation.	$a = \frac{y - y_1}{x - x_1} = \frac{3 - (-7)}{-5 - (-3)} = \frac{3 + 7}{-5 + 3} = \frac{10}{-2} = -5$ Use point D(-3; -7) and substitute into $a(x - x_1) = y - y_1$ $-5(x - (-3)) = y - (-7)$ $-5(x + 3) = y + 7$ $-5x - 15 - 7 = y + 7 - 7$ $-5x - 22 = y$ OR, using point F(-5; 3): $-5(x - (-5)) = y - 3$ $-5(x + 5) = y - 3$ $-5x - 25 + 3 = y - 3 + 3$ $-5x - 22 = y$
2) Does the point G(-2; 3) lie on the line having equation $y = \frac{3}{2}x - 4$? Note: Substitute $x = -2$ into the RHS of the equation and $y = 3$ into the LHS of the equation.	$\text{LHS} = y = 3$ $\text{RHS} = \frac{3}{2}x - 4 = \frac{3}{2}(-2) - 4 = -3 - 4 = -7$ $\text{LHS} \neq \text{RHS}$, So G does not lie on the line.

Exercise 8.3

1) Find the equation of the line passing through the points B(-4; 9) and T(-6; 5).

2) Does the point G(-2; 3) lie on the line having equation $y = \frac{3}{2}x - 4$.

§ 8.4 CHANGING THE FORM OF A LINEAR EQUATION

✦ Equations of straight line graphs can be written in different forms. These can be converted from one form to another.

EXAMPLE	SOLUTION
1) Write the equation $y = \frac{-1}{2}x - \frac{1}{4}$ in the form $Ax + By + C = 0$	The HCF of 2 and 4 is 4. So multiply both sides by 4. $4y = 4\left(\frac{-1}{2}x - \frac{1}{4}\right)$ $4y = -2x - 1$ $4y + 2x + 1 = -2x - 1 + 2x + 1$ $4y + 2x + 1 = 0$
2) Write the equation $3y - 2x + 9 = 0$ in the form $y = ax + q$	To have y alone on one side, we must divide each term on both sides by 3. $\frac{3y}{3} - \frac{2x}{3} + \frac{9}{3} = \frac{0}{3}$ $y - \frac{2x}{3} + 3 = 0$ $y - \frac{2x}{3} = -3$ $y - \frac{2x}{3} + \frac{2x}{3} = -3 + \frac{2x}{3}$ $y = \frac{2x}{3} - 3$

Exercise 8.4

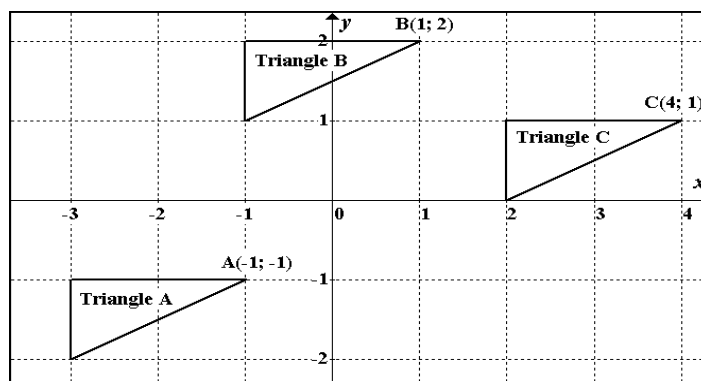
1) Write the equation $y = \frac{2}{3}x - \frac{5}{6}$ in the form $Ax + By + C = 0$

2) Write the equation $6x - 4y - 8 = 0$ in the form $y = ax + q$.

§ 8.5 TRANSLATING POLYGONS

- ✧ Translating a polygon changes its position, but not its size or shape.
- ✧ A translation is written $(x; y)$, and acts on every point in the polygon.

EXAMPLE:



In the above graph:

Triangle A has moved 2 units to the right and 3 units up to triangle B. Translation $(2; 3)$.

The point $(-1; -1)$ moved to the point $(-1 + 2; -1 + 3) = (1; 2)$.

Triangle B moved 3 units to the right and 1 unit down to triangle C. Translation $(3; -1)$.

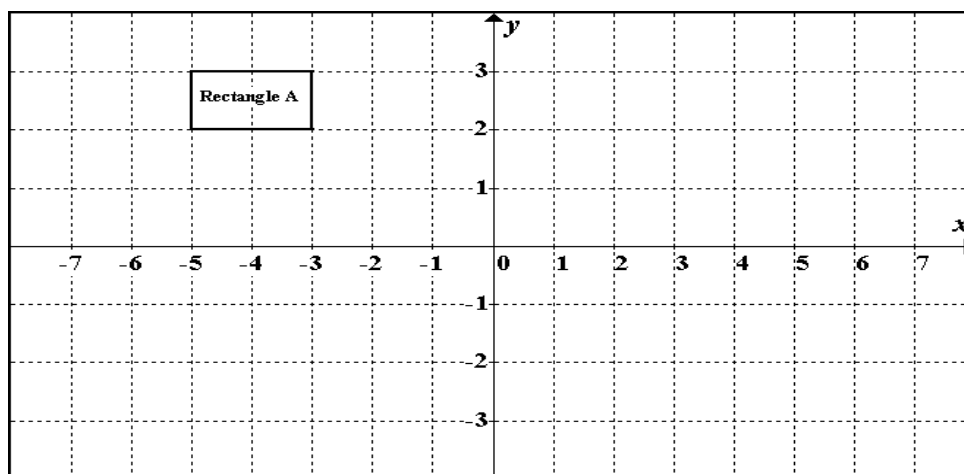
The point $(1; 2)$ has moved to the point $(1 + 3; 2 - 1) = (4; 1)$

EXAMPLE	SOLUTION
Find the translation as triangle A in the above diagram moves to triangle C. $A(-1; -1)$ moves to $C(4; 1)$.	Let the translation be $(x; y)$. $\begin{aligned} -1 + x &= 4 & \text{and} & & -1 + y &= 1 \\ -1 + x + 1 &= 4 + 1 & & & -1 + y + 1 &= 1 + 1 \\ x &= 5 & & & y &= 2 \end{aligned}$ The translation is $(5; 2)$

Exercise 8.5

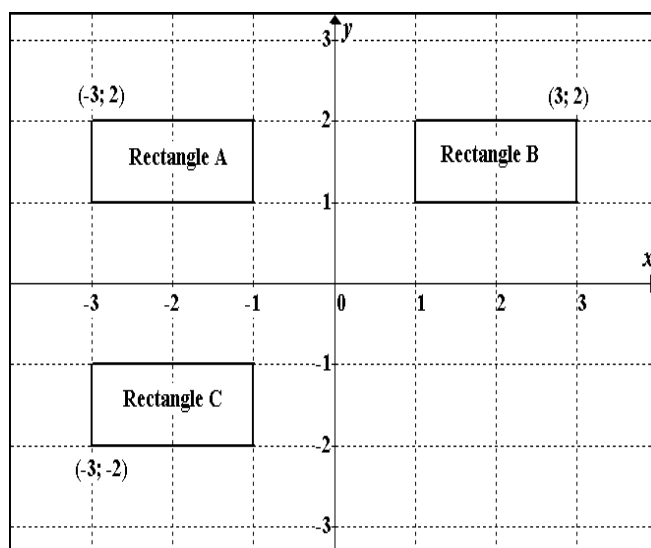
On the grid, draw the image of Rectangle A after each of the following translations:

- 1) Rectangle B after the translation $(6; -4)$
- 2) Rectangle C after the translation $(4; -2)$
- 3) Rectangle D after the translation $(-2; -5)$
- 4) Rectangle E after the translation $(8; -6)$



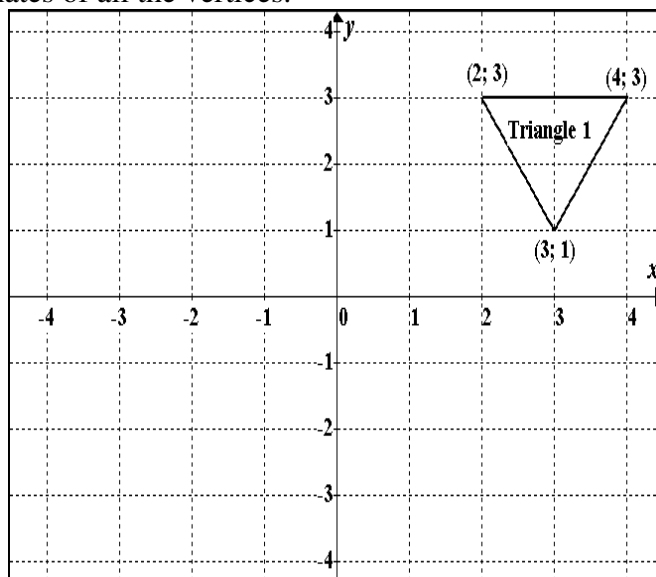
§ 8.6 REFLECTION OF A POLYGON ABOUT THE x -AXIS AND ABOUT THE y -AXIS

- ✦ Reflecting a polygon about the x -axis changes **the sign of the y -coordinate** of each point on the polygon. In the graph below, $(-3; 2)$ on Rectangle A reflects about the x -axis to $(-3; -2)$ on Rectangle B.
- ✦ Reflecting a polygon about the y -axis changes **the sign of the x -coordinate** of each point on the polygon. In the graph, $(-3; 2)$ on Rectangle A reflects about the y -axis to $(3; 2)$ on Rectangle B.



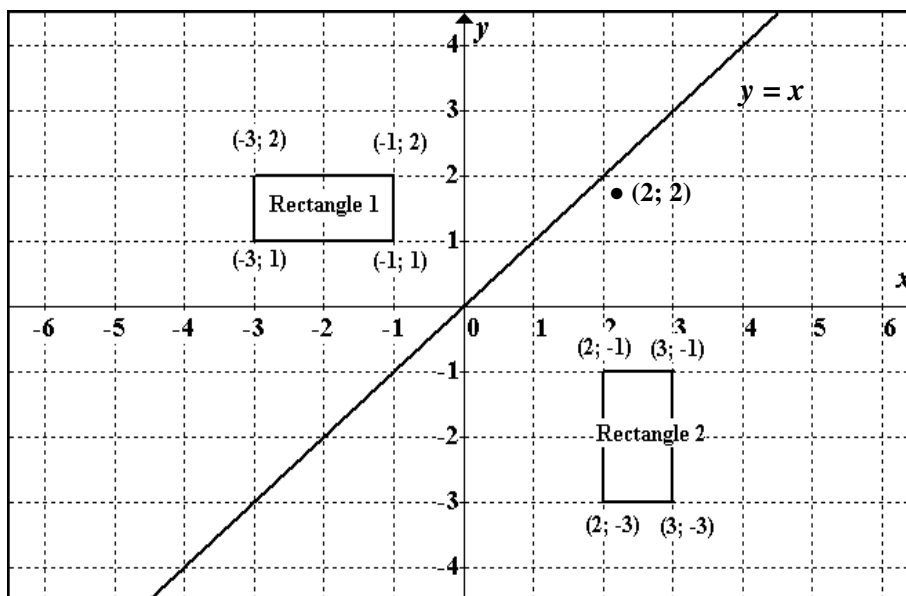
Exercise 8.6

- 1) On the grid:
 - a) draw the reflection of Triangle 1 in the x -axis (Triangle 2)
 - b) the reflection of Triangle 1 in the y -axis (Triangle 3).
- 2) Show the coordinates of all the vertices.



§ 8.7 REFLECTION OF A POLYGON ABOUT THE LINE $y = x$.

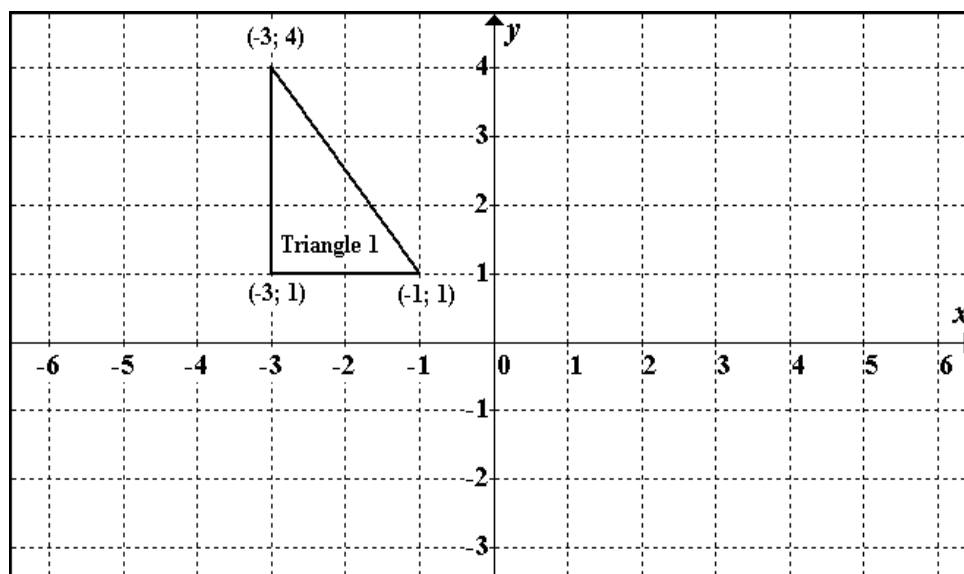
- ✧ Reflecting a polygon in the line $y = x$, reverses the x and y -coordinates of each point on the polygon. In the graph, $(-3; 2)$ on Rectangle 1 reflects in the x -axis to $(2; -3)$ on Rectangle 2.



Exercise 8.7

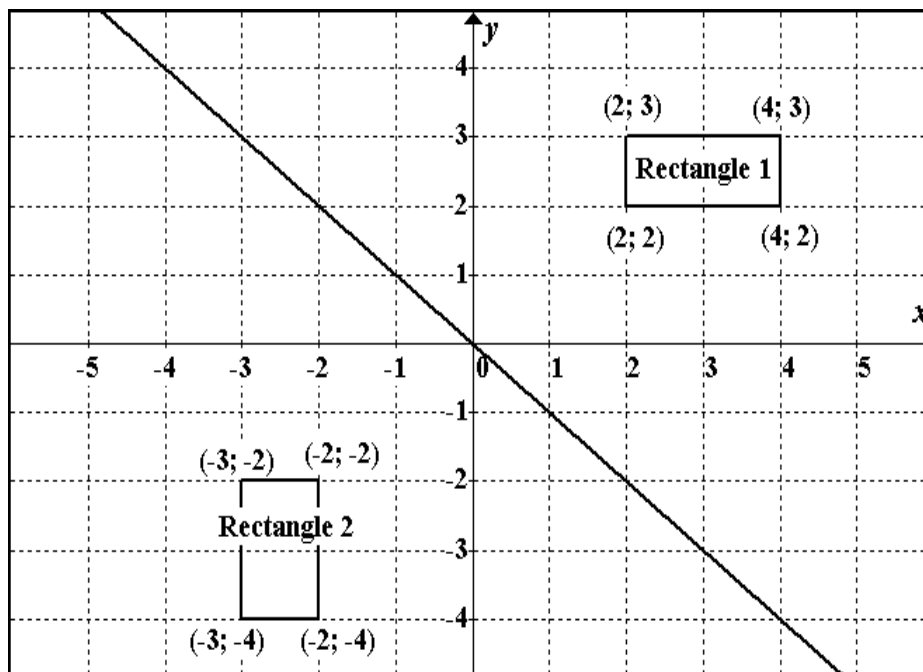
- 1) Draw the line $y = x$.
- 2) Reflect triangle 1 in the line $y = x$ and draw it on the grid (Triangle 2). Show the coordinates of each of the vertices.

b



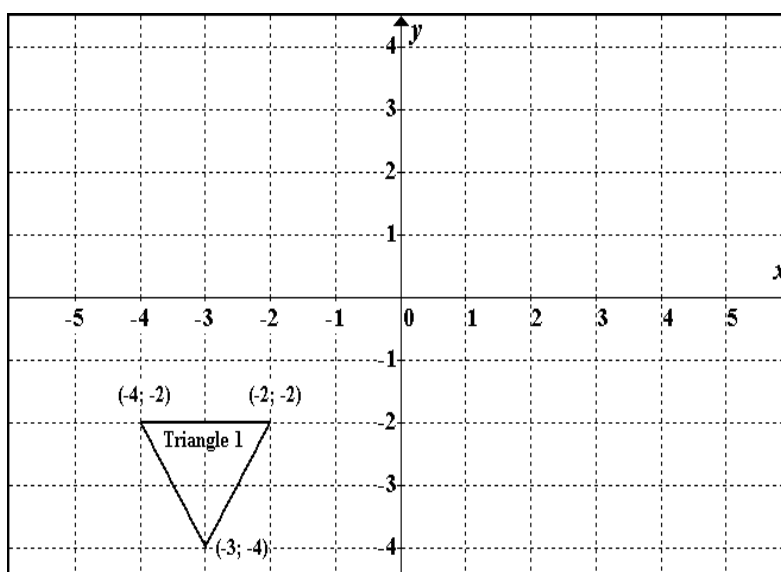
§ 8.8 REFLECTION OF A POLYGON ABOUT THE LINE $y = -x$

- ✧ Reflecting a polygon in the line $y = -x$, reverses the x and y -coordinates of each point on the polygon and changes the signs of both. In the graph, $(2; 3)$ on Rectangle 1 reflects in the line $y = -x$ to $(-3; -2)$ on Rectangle 2.



Exercise 8.8

- 1) Draw the line $y = -x$.
- 2) Reflect the polygon (Triangle 1) in the line $y = -x$ and draw it on the grid (Triangle 2). Show the coordinates of each of the vertices.



CHAPTER 9

Trigonometry

In this chapter you will:

- Name sides and angles in a right-angled triangle
- Practise using the theorem of Pythagoras
- Use the definitions for the three trig ratios
- Use the sin and cos ratios to find the length of a side
- Use the sin and cos ratios to find the length of the hypotenuse
- Use the tan ratio to find the length of a side
- Use the sin ratio to find the size of an angle
- Use the cos ratio to find the size of an angle
- Use the tan ratio to find the size of an angle
- Draw the graph of the sin function
- Draw the graph of $y = a \sin x$
- Draw the graph of $y = \sin x + q$
- Draw the graph of $y = a \sin x + q$
- Draw the graph of the cos function
- Draw the graph of $y = a \cos x + q$
- Draw the graph of the tan function.

This chapter covers material from Topic 3: Space, Shape and Orientation

SUBJECT OUTCOME 3.4:

Solve problems by constructing and interpreting geometrical and trigonometric models

Learning Outcome 5: Know and use the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$

Learning Outcome 6: Sketch the sin, cos and tan functions and discuss their periodicity.

§ 9.1 NAMING SIDES AND ANGLES IN A RIGHT-ANGLED TRIANGLE

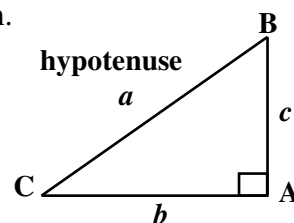
✦ The sides of a triangle are named from the angles opposite them.

In $\triangle ABC$, BC, the side BC opposite angle A ($\hat{A}$) is called a .

the side AC opposite angle B ($\hat{B}$) is called b .

the side AB opposite angle C ($\hat{C}$) is called c .

✦ The side opposite the right angle, $\hat{A}$, is the **hypotenuse**.

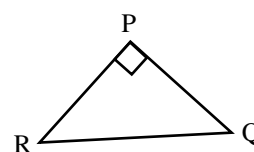
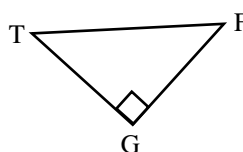


EXAMPLE	SOLUTION
1) Label the sides in $\triangle ABC$ which is right-angled at A, and $\triangle DEF$ which is right-angled at F. Mark the hypotenuse in each triangle.	
2) Label each vertex and hypotenuse on the following two triangles:	

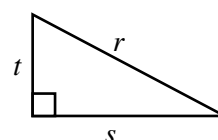
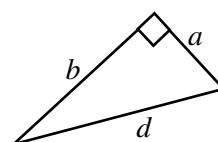
NOTE: One vertex, many vertices.

Exercise 9.1

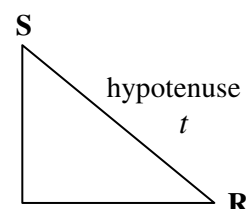
- 1) Label
 - a) Each of the sides in each of the triangles
 - b) Each hypotenuse.



- 2) For each triangle label the vertices and the hypotenuse:



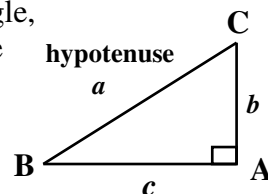
- 3) For following triangle fill in the labels for the angles and sides that are missing.



§ 9.2 THE THEOREM OF PYTHAGORAS

✦ The **Theorem of Pythagoras** states that in a right-angled triangle, the hypotenuse squared is equal to the sum of the squares of the other two sides.

In other words, $a^2 = b^2 + c^2$ only in a right - angled triangle.

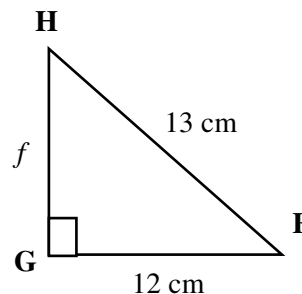


EXAMPLE	SOLUTION
Calculate r in $\triangle PQR$: Note: this triangle is right - angled.	Because $\triangle PQR$ is right angled, $q^2 = p^2 + r^2 \text{ (Pythagoras)}$ $25^2 = 7^2 + r^2$ $625 = 49 + r^2$ $625 - 49 = r^2$ $576 = r^2$ $\sqrt{576} = r$ $24 \text{ cm} = r$

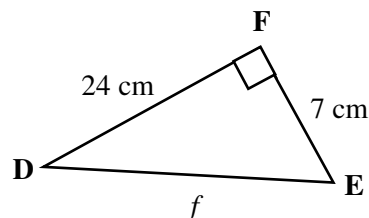
Exercise 9.2

Use the theorem of Pythagoras to:

- 1) find the length of side f in $\triangle FGH$



- 2) calculate the length of side f in $\triangle DEF$

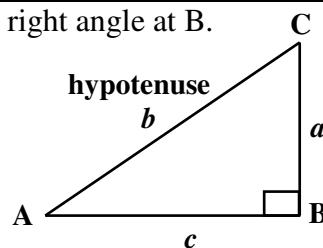


§ 9.3 THREE TRIG RATIOS IN A RIGHT-ANGLED TRIANGLE

✦ In triangle ABC, **b** is the **hypotenuse**, since it is opposite the right angle at B.

✦ For $\hat{A}$: **a** is the **opposite** side (opposite $\hat{A}$)
c is the **adjacent** side (next to $\hat{A}$)

✦ For $\hat{C}$: **c** is the **opposite** side (opposite $\hat{C}$)
a is the **adjacent** side (next to $\hat{C}$)



✦ We define the three trigonometric ratios *sin*, *cos* and *tan* as follows:

$$\sin \hat{A} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{b}$$

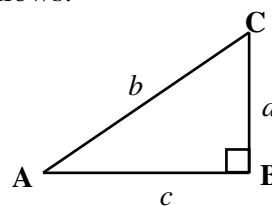
$$\sin \hat{C} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{c}{b}$$

$$\cos \hat{A} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{c}{b}$$

$$\cos \hat{C} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{b}$$

$$\tan \hat{A} = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{c}$$

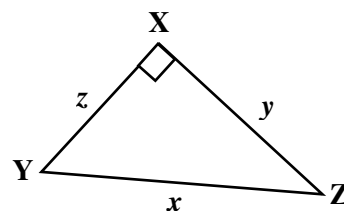
$$\tan \hat{C} = \frac{\text{opposite}}{\text{adjacent}} = \frac{c}{a}$$



Exercise 9.3

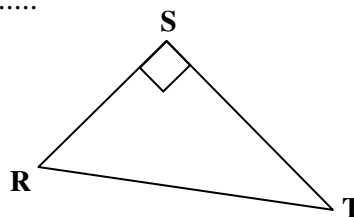
1) Use $\triangle XYZ$ to write down:

- the side adjacent to $\hat{Z}$
- the side opposite $\hat{Y}$
- the hypotenuse
- the side adjacent to $\hat{Y}$
- the side opposite $\hat{Z}$



2)

a) Label the sides of $\triangle RST$.



b) Use $\triangle RST$ to write down:

i) $\tan \hat{R} = \dots\dots\dots$

ii) $\tan \hat{T} = \dots\dots\dots$

iii) $\sin \hat{R} = \dots\dots\dots$

iv) $\sin \hat{T} = \dots\dots\dots$

iv) $\cos \hat{R} = \dots\dots\dots$

v) $\cos \hat{T} = \dots\dots\dots$

3) Draw a triangle in which $\cos \hat{P} = \frac{b}{t}$

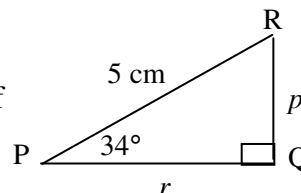
§ 9.4 USING THE SIN AND COS RATIOS TO FIND THE LENGTH OF A SIDE

EXAMPLE

Given: Hypotenuse = 5 cm $\hat{P} = 34^\circ$

Use $\triangle PQR$ to find the length, correct to 2 decimal places of

- 1) sides p and
- 2) side r ,



SOLUTION

- 1) Side p is opposite $\hat{P}$, so we use the sin ratio which contains both opposite and hypotenuse:

$$\begin{aligned}\sin 34^\circ &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{p}{5}\end{aligned}$$

$$5 \times \sin 34^\circ = 5 \times \frac{p}{5}$$

$$2,795 \dots \text{ cm} = p$$

So $p = 2,80$ (correct to 2 decimal places)

Key sequence	Display
5 sin 34 =	2.795

- 2) Side r is adjacent to $\hat{P}$, so use the cos ratio which contains both adjacent and hypotenuse:

$$\begin{aligned}\cos 34^\circ &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{r}{5}\end{aligned}$$

$$5 \times \cos 34^\circ = 5 \times \frac{r}{5}$$

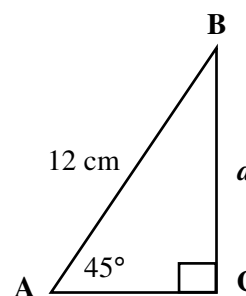
So $r = 4,15$ (correct to 2 decimal places)

Key sequence	Display
5 cos 34 =	4.145

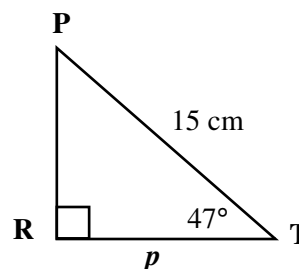
Exercise 9.4

Give answers correct to 2 decimal places:

- 1) Use $\triangle ABC$ to find the length of side a .



- 2) In $\triangle PTR$, calculate the length of side p .

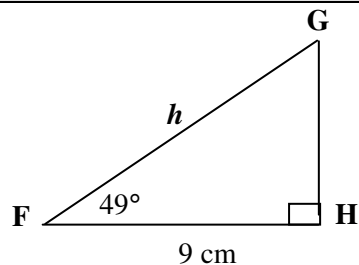


§ 9.5 USING THE SIN AND COS RATIOS TO FIND THE LENGTH OF THE HYPOTENUSE

EXAMPLE

Given: $\hat{F} = 49^\circ$ and $g = 9$ cm, adjacent to $\hat{F}$

Find the length of the hypotenuse h of $\triangle FGH$, correct to the nearest whole cm



SOLUTION

Side h is the hypotenuse. Use the cos ratio which contains both adjacent and hypotenuse:

$$\begin{aligned}\cos 49^\circ &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{9}{h}\end{aligned}$$

$$h \times \cos 49^\circ = h \times \frac{9}{h}$$

$$h \cos 49^\circ = 9$$

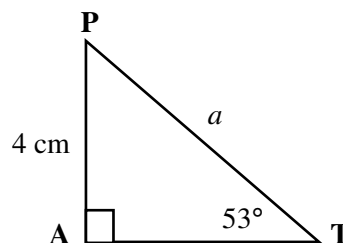
$$\frac{h \cos 49^\circ}{\cos 49^\circ} = \frac{9}{\cos 49^\circ}$$

$$h = 14 \text{ cm (correct to the nearest cm)}$$

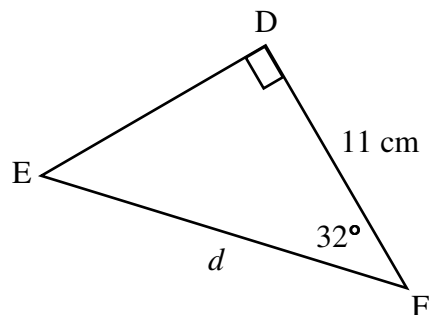
Key sequence	Display
9 $\div$ $\cos$ 49 $=$	13.71....

Exercise 9.5

- 1) Calculate the length of g , the hypotenuse of $\triangle PTA$.
Answer to the nearest whole cm.



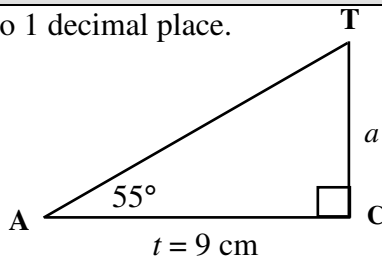
- 2) Calculate the length of hypotenuse d in $\triangle DEF$, correct to the nearest whole cm.



§ 9.6 USING THE TAN RATIO TO FIND THE LENGTH OF A SIDE

EXAMPLE

Use $\triangle CAT$ to find the length of side a , correct to 1 decimal place.



SOLUTION

Side a is opposite to the given $\hat{A}$ and side t is adjacent to the given $\hat{A}$, so we use the tan ratio which contains both opposite and adjacent.

$$\begin{aligned}\tan 55^\circ &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{a}{9}\end{aligned}$$

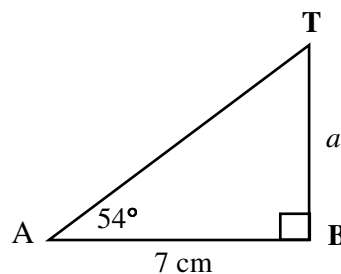
$$9 \times \tan 55^\circ = 9 \times \frac{a}{9}$$

$$12,9 \text{ cm} = a$$

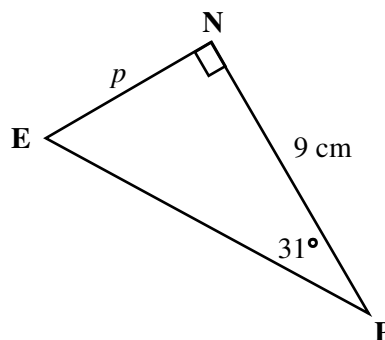
Key sequence	Display
9 tan 55 =	12.853....

Exercise 9.6

- 1) Find the length of side a in $\triangle BAT$, correct to one decimal place.



- 2) Calculate the length of side p in $\triangle PEN$, correct to one decimal place.



§ 9.7 USING THE SIN RATIO TO FIND THE SIZE OF AN ANGLE

✦ We can use the three trig ratios to calculate the size of an angle in a right-angled triangle, by using the keys on the calculator labelled $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$

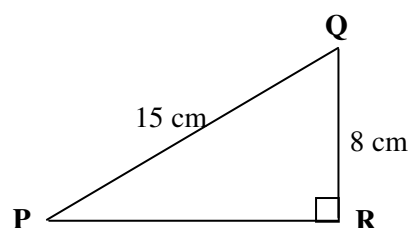
Note:

- to use $\sin^{-1}$ key in $\boxed{\text{shift}} \boxed{\sin}$
- to use $\cos^{-1}$ key in $\boxed{\text{shift}} \boxed{\cos}$
- to use $\tan^{-1}$ key in $\boxed{\text{shift}} \boxed{\tan}$

EXAMPLE

In the right-angled $\triangle PQR$ calculate:

- 1) the size of $\hat{P}$, correct to one decimal place
- 2) the size of $\hat{Q}$.



SOLUTION

Given: the length of the hypotenuse and of the side opposite to $\hat{P}$, so use the sin ratio

$$\begin{aligned} 1) \sin \hat{P} &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{8}{15} \end{aligned}$$

Key sequence	Display
$\boxed{\sin^{-1}} \boxed{8} \boxed{\div} \boxed{15} \boxed{=}$	32.23.....

$$\hat{P} = 32,2^\circ \text{ (correct to one decimal place)}$$

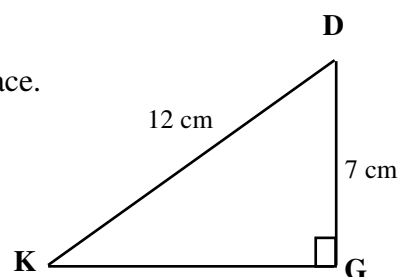
- 2) The three angles of a triangle add up to 180° , so $32,2^\circ + 90^\circ + \hat{Q} = 180^\circ$

$$122,2^\circ + \hat{Q} = 180^\circ$$

$$\begin{aligned} \hat{Q} &= 180^\circ - 122,2^\circ \\ &= 57,8^\circ \end{aligned}$$

Exercise 9.7

- 1) Use $\triangle DKG$ to find the size of $\hat{K}$ correct to 1 decimal place.



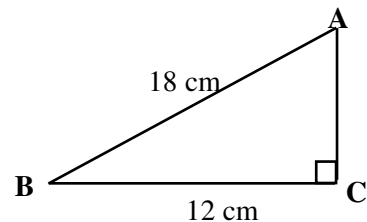
- 2) Find the size of $\hat{D}$ correct to one decimal place.

§9.8 USING THE COS RATIO TO FIND THE SIZE OF AN ANGLE

EXAMPLE

In the right-angled triangle ABC, calculate:

- 1) the size of $\hat{B}$ correct to the nearest whole degree
- 2) the size of $\hat{A}$



SOLUTION

- 1) Given: the lengths of the hypotenuse and the side adjacent to angle B, so use the cos ratio.

$$\begin{aligned}\cos \hat{B} &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{12}{18}\end{aligned}$$

$$\hat{B} = 49^\circ \text{ (correct to the nearest degree)}$$

Key sequence	Display
$\boxed{\cos^{-1}} \boxed{12} \boxed{\div} \boxed{18} \boxed{=}$	48.18.....

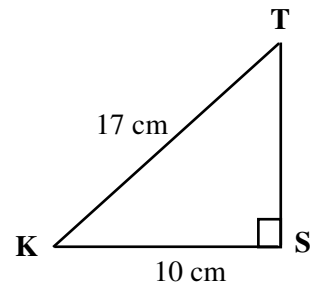
- 2) The three angles of a triangle add to 180° , so $\hat{A} + 90^\circ + 49^\circ = 180^\circ$

$$\hat{A} + 139^\circ = 180^\circ$$

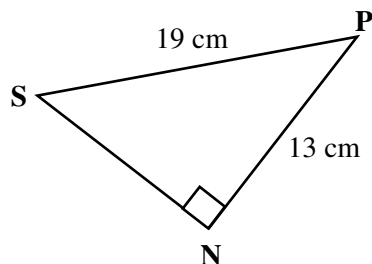
$$\begin{aligned}\hat{A} &= 180^\circ - 139^\circ \\ &= 41^\circ\end{aligned}$$

Exercise 9.8

- 1) Calculate the sizes of $\hat{K}$ and $\hat{T}$ in triangle KST, correct to the nearest whole degree.



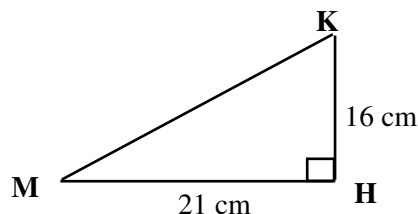
- 2) Use $\triangle TRS$ to find the sizes of $\hat{P}$ and $\hat{S}$, correct to one decimal place.



§9.9 USING THE TAN RATIO TO FIND THE SIZE OF AN ANGLE

EXAMPLE

In $\triangle MHK$, calculate the sizes of $\hat{M}$ and $\hat{K}$, correct to the nearest whole degree.



SOLUTION

Given: the length of the opposite and adjacent sides to $\hat{K}$, so use the tan ratio.

$$\tan \hat{K} = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{21}{16}$$

$$\hat{K} = 53^\circ$$

Key sequence	Display
$\tan^{-1}$ 21 $\div$ 16 $=$	52.69.....

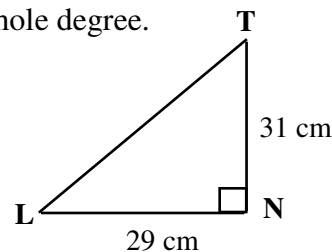
Since the angles of a triangle add up to 180° , $\hat{M} + 90^\circ + 53^\circ = 180^\circ$

$$\hat{M} + 143^\circ = 180^\circ$$

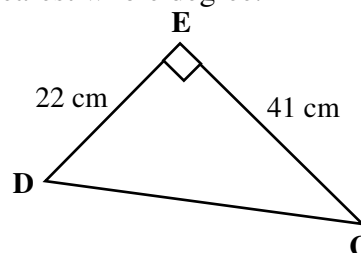
$$\begin{aligned}\hat{M} &= 180^\circ - 143^\circ \\ &= 37^\circ\end{aligned}$$

Exercise 9.9

- 1) Calculate the size of $\hat{L}$ and $\hat{T}$ in $\triangle LNT$, correct to the nearest whole degree.



- 2) In $\triangle CDE$, find the size of $\hat{C}$ and $\hat{D}$, both correct to the nearest whole degree.



§ 9.10 THE GRAPH OF THE SIN FUNCTION

- ✦ The x -axis is measured in degrees and the y -axis in units.
- ✦ The **period** of a trig graph is the number of degrees (measured along the x -axis) necessary to complete one pattern.
- ✦ The **amplitude** is the positive distance between the axis and the maximum y value on the graph.

EXAMPLE

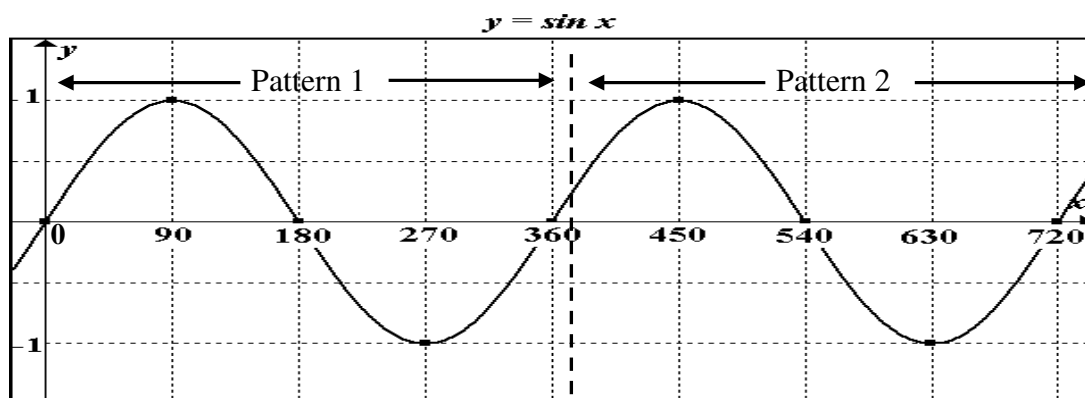
- 1) Complete the table using a calculator
- 2) Use the table to draw the graph of $y = \sin x$
- 3) Write down the period and amplitude of the graph.

SOLUTION

1)

x	0°	90°	180°	270°	360°	450°	540°	630°	720°
$y = \sin x$	0	1	0	-1	0	1	0	-1	0

2)

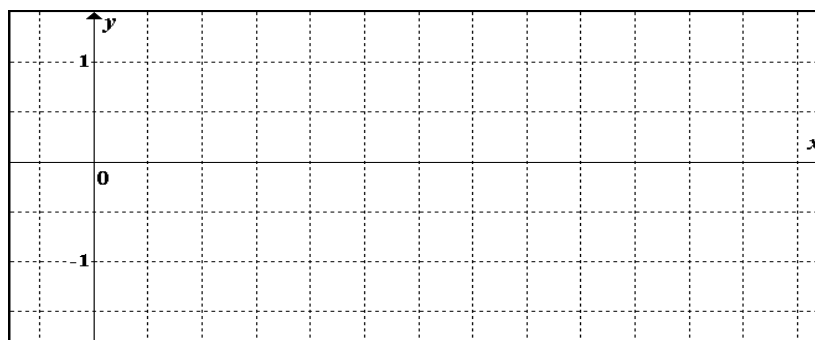


- 3) One graph pattern is completed in 360° . $y = \sin x$ has a **period** = 360° .
The graph shows two complete patterns. More x values to the left and right would show that this pattern is repeated endlessly, in both the negative and the positive directions.
The distance between the x -axis and the maximum y value is 1, so the **amplitude** = 1.

Exercise 9.10

- 1) Use a calculator to complete the table. 2) Draw the graph of $y = \sin x$ for $0^\circ \leq x \leq 180^\circ$.

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
$y = \sin x$									



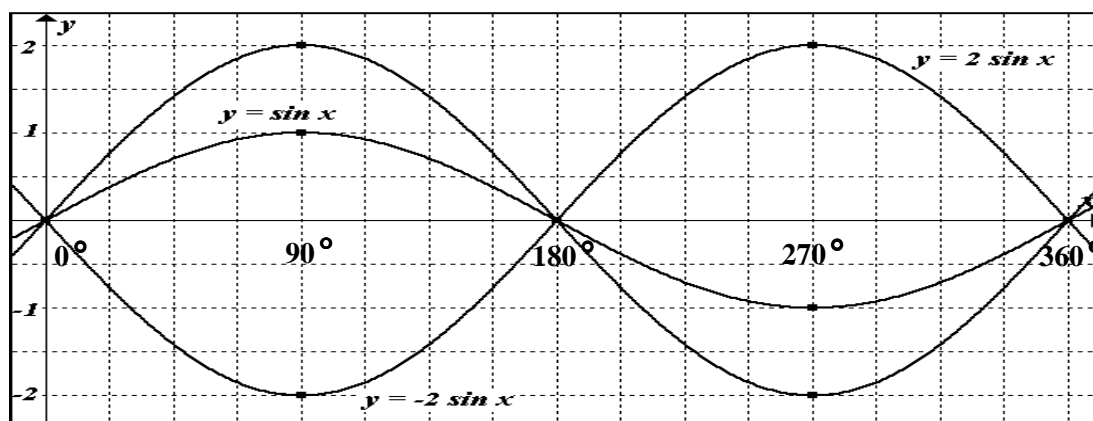
§ 9.11 THE GRAPH OF $y = a \sin x$

EXAMPLE

Complete the table and on the same set of axes draw graphs of $y = 2\sin x$, $y = \sin x$ and $y = -2\sin x$.

SOLUTION

x	0°	90°	180°	270°	360°
$y = 2 \sin x$	0	2	0	-2	0
$y = \sin x$	0	1	0	-1	0
$y = -2 \sin x$	0	-2	0	2	0



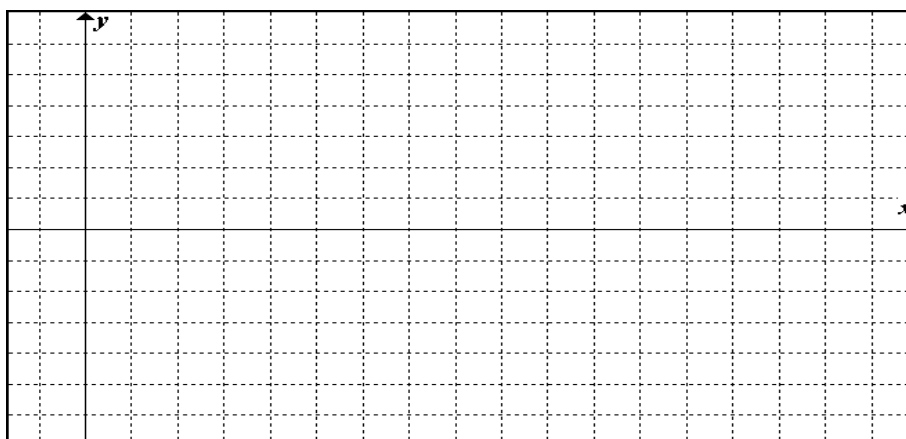
Note:

- Graphs have period 360° . The value of a does not affect the period of $y = a \sin x$.
- $a > 0$ and $y > 0$, the graph lies in quadrants 1 and 2
- $a > 0$ and $y < 0$, the graph lies in quadrants 3 and 4
- $a < 0$ and $y < 0$, the graph lies in quadrants 1 and 2
- $a < 0$ and $y > 0$, the graph lies in quadrants 3 and 4
- The positive numerical value of a gives the amplitude of the graph of $y = a \sin x$.

Exercise 9.11

Complete the table, draw and label the graphs of $y = 3\sin x$ and $y = -\sin x$ for $0 \leq x \leq 360^\circ$.

x	0°	30°	90°	150°	180°	210°	270°	330°	360°
$y = 3 \sin x$									
$y = -\sin x$									



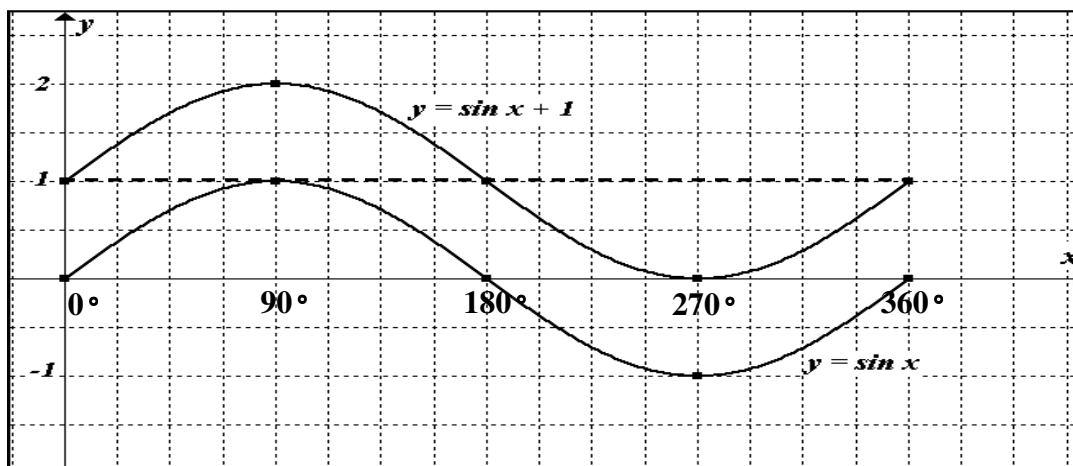
§ 9.12 THE GRAPH OF $y = \sin x + q$

EXAMPLE

Complete the table and on the same set of axes draw the graphs of $y = \sin x$ and $y = \sin x + 1$, where $0 \leq x \leq 360^\circ$.

SOLUTION

x	0°	90°	180°	270°	360°
$y = \sin x$	0	1	0	-1	0
$y = \sin x + 1$	1	2	1	0	1



Note:

For $y = \sin x + 1$:

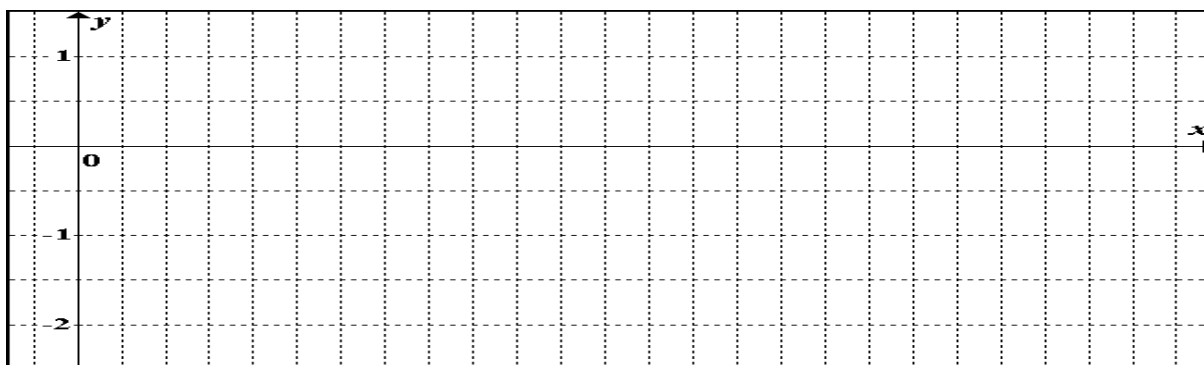
- The period of this graph is 360° , so q does not affect the period.
- The graph rotates about the line $y = 1$ (dotted line). The amplitude is still 1.
- The effect of $q = 1$ is to raise the graph vertically through 1 unit.

Exercise 9.12

Complete the following table and on the same axes draw the graphs of $y = \sin x$ and

$y = \sin x - 1$, where $0 \leq x \leq 360^\circ$.

x	0°	30°	90°	150°	180°	210°	270°	330°	360°
$y = \sin x$									
$y = \sin x - 1$									



§ 9.13 THE GRAPH OF $y = a \sin x + q$

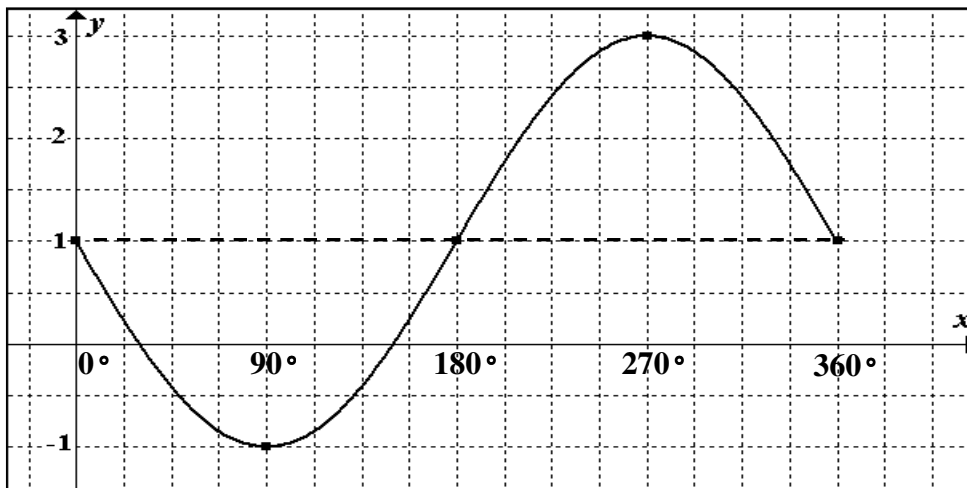
EXAMPLE

Complete the table and then draw the graph of $y = -2 \sin x + 1$, where $0 \leq x \leq 360^\circ$.

SOLUTION

x	0°	90°	180°	270°	360°
$y = -2 \sin x + 1$	1	-1	1	3	1

$$y = -2 \sin x + 1$$



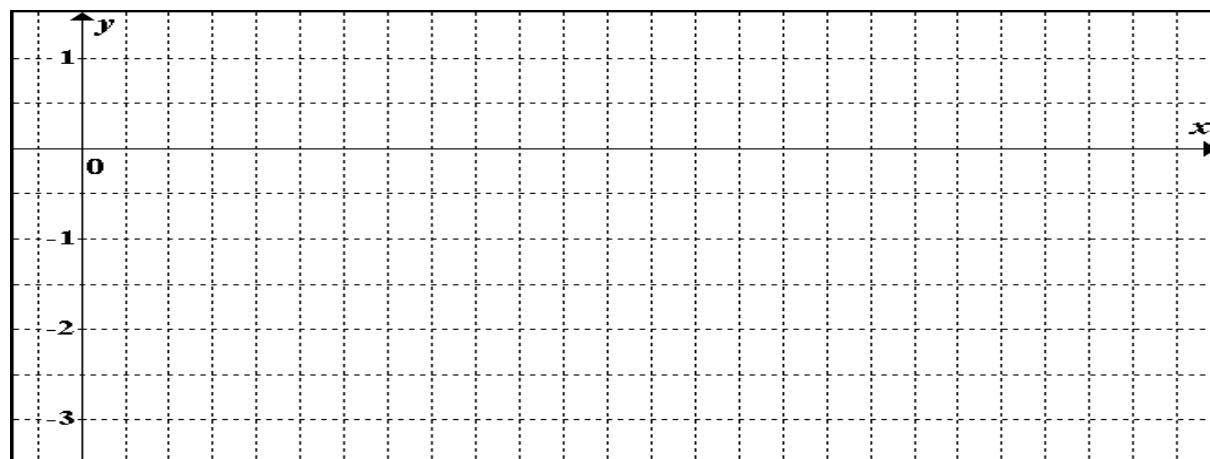
Note:

- The period is 360°
- The graph of $y = -2 \sin x + 1$ rotates about the line $y = 1$ (dotted line)
- The amplitude is 2

Exercise 9.13

Complete the table and draw the graph of $y = 2 \sin x - 1$, $0 \leq x \leq 360^\circ$.

x	0°	30°	90°	150°	180°	210°	270°	330°	360°
$y = \sin x$									
$y = 2 \sin x$									
$y = 2 \sin x - 1$									



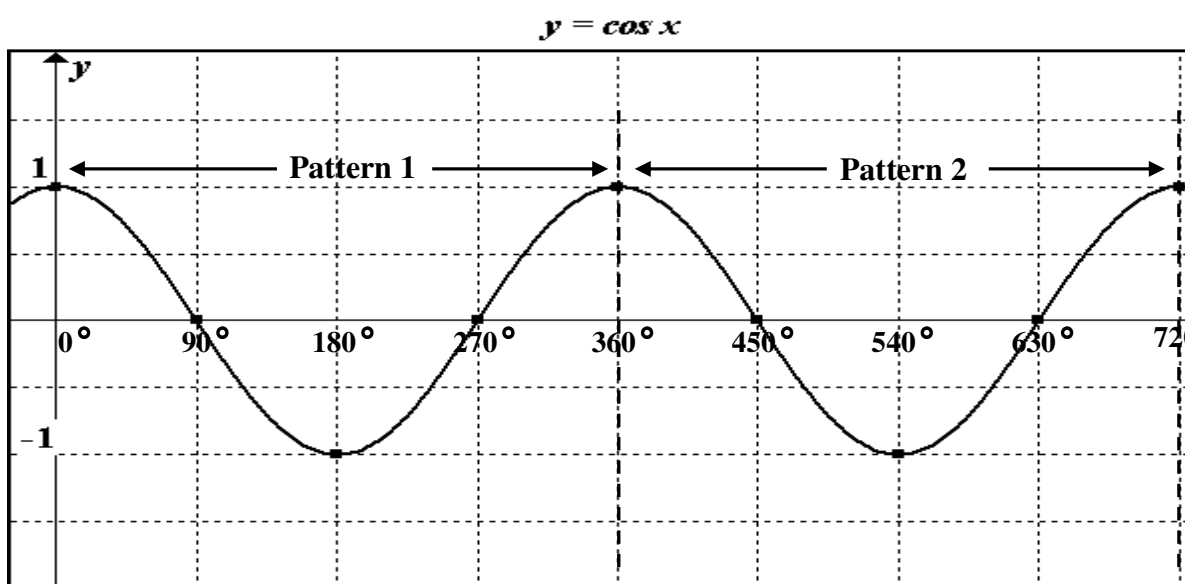
§ 9.14 THE GRAPH OF THE COS FUNCTION

EXAMPLE

Complete the table using a calculator. Use the table to draw the graph of $y = \cos x$

SOLUTION

x	0°	90°	180°	270°	360°	450°	540°	630°	720°
$y = \cos x$	1	0	-1	0	1	0	-1	0	1

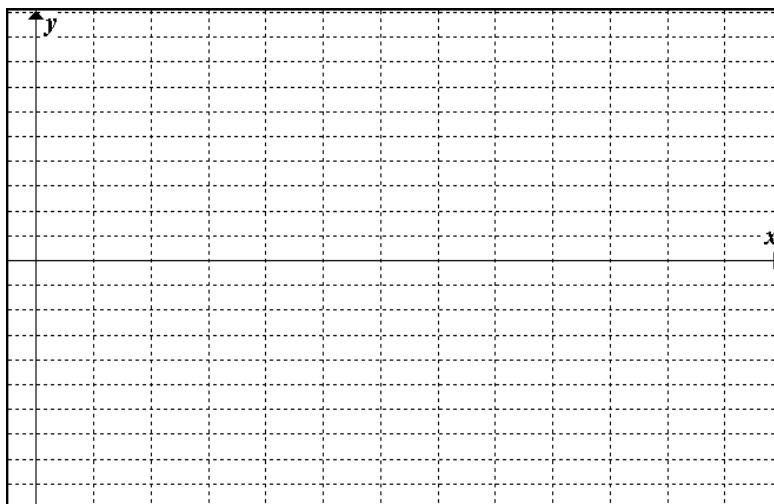


- ✦ One graph pattern is completed in 360° . $y = \cos x$ has **period** = 360° . This graph shows two complete patterns. Taking x values to the left and right of those above would show that this pattern is repeated endlessly, in both the negative and the positive directions.
- ✦ The distance between the y -axis and the maximum y value is 1, so the **amplitude** = 1.

Exercise 9.14

1) Use a calculator to complete the table. 2) Draw the graph of $y = \cos x$ for $0^\circ \leq x \leq 180^\circ$.

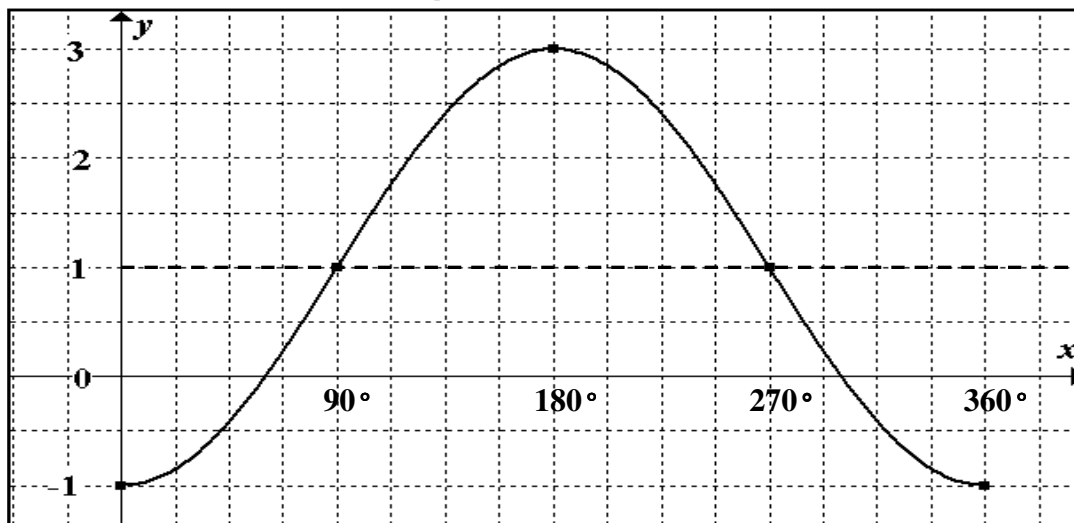
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
$y = \cos x$									



§ 9.15 THE GRAPH OF $y = a \cos x + q$ **EXAMPLE**Complete the table and draw the graph of $y = -2 \cos x + 1$, $0^\circ \leq x \leq 360^\circ$.**SOLUTION**

x	0°	90°	180°	270°	360°
$y = -2 \cos x + 1$	-1	1	3	1	-1

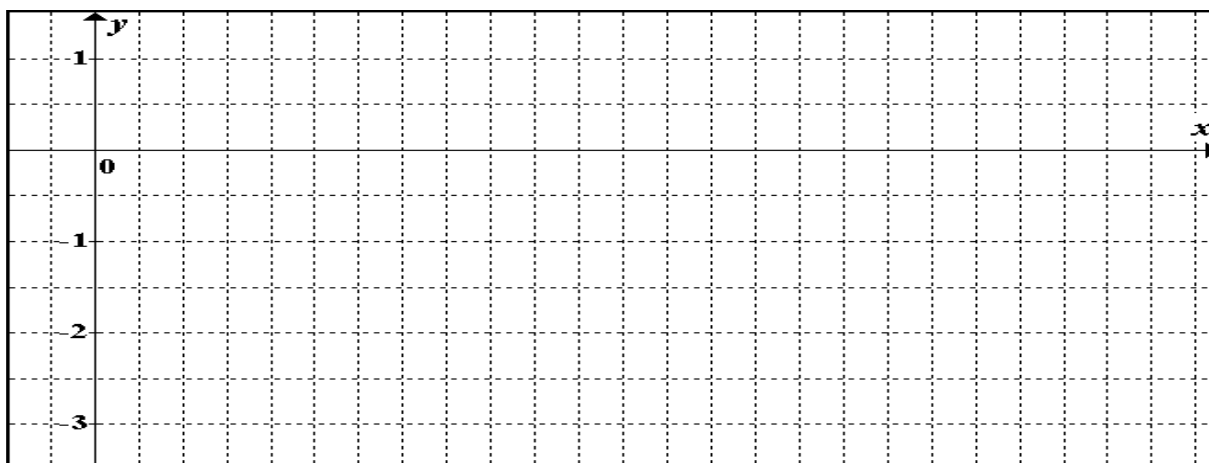
$$y = -2 \cos x + 1$$

**Note:**

- The period is 360°
- The graph of $y = -2 \cos x + 1$ rotates about $y = 1$ (dotted line)
- The amplitude is 2

Exercise 9.15Complete the table and draw the graphs of $y = 2 \cos x - 1$, $0 \leq x \leq 360^\circ$.

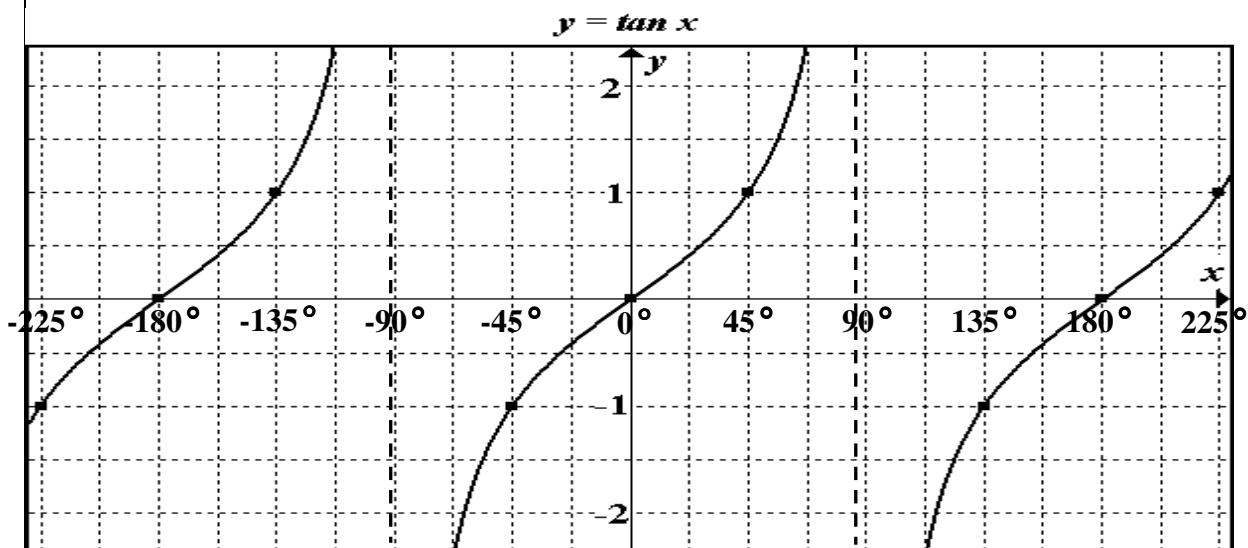
x	0°	60°	90°	120°	180°	240°	270°	300°	360°
$y = \cos x$									
$y = 2 \cos x - 1$									



§ 9.16 THE GRAPH OF THE TAN FUNCTION

EXAMPLE1) Complete the table using a calculator. 2) Draw the graph of $y = \tan x$, $-225^\circ \leq x \leq 225^\circ$.**SOLUTION**

x	-225°	-180°	-135°	-45°	0°	45°	135°	180°	225°
$y = \tan x$	-1	0	1	-1	0	1	-1	0	1

**Note:**

- The tan function has **no amplitude**
- The period of the graph is 180° , and $x = -90^\circ$ and $x = 90^\circ$ are asymptotes
- Points whose x -coordinates are multiples of 45° are added to show the shape clearly

Exercise 9.16Complete the table, and draw and label graphs for $-90^\circ \leq x \leq 90^\circ$

x	-90°	-45°	0°	45°	90°
$y = \tan x$					
$y = \tan x + 1$					
$y = \tan x - 1$					

